

# EXPONENTIAL FUNCTIONS, COOLING CURVES, VOLUME/AREA RATIOS

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## Exponential Functions

Last week we discussed that exponential functions occur when the rate of change of a quantity is proportional to the instantaneous magnitude of that quantity. In last week's activity, we saw that the rate of M & M decay was proportional to the instantaneous number of M & Ms. In other words, the rate of decay (number of M & Ms lost/turn) was proportional to the number of M & Ms at the beginning of the turn. If say, you had 1000 M & Ms, you would expect to lose 500 of them in the turn; if though you had only 100 M & Ms, you would lose only about 50. In the language of last week, we would write :

$$\frac{\Delta M}{\Delta t} = \frac{-1}{2} M$$

in this case, M represents number of M & Ms at a given time t,  $\Delta M$  is the change in the number of M&Ms, and  $\Delta M/\Delta t$  means the number of M&Ms lost per turn. The minus sign on the right tells you that we are losing candies each turn. (A positive constant on the right means that the quantity M is increasing exponentially in time.)

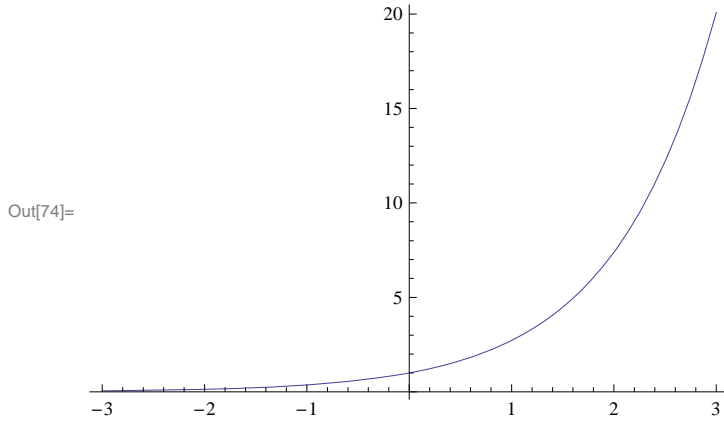
The M & M process we studied is very useful in demonstrating radioactive decay. Remember that atomic nuclei consist of protons and neutrons; there are processes that occur in nuclei in which a given nucleus emits either matter or energy. In the process, the nature of the pre - existing (or "parent") nucleus changes its nature and changes (or "decays") into a new type of nucleus, called the "daughter" product. The number of decays depends on how many nuclei there are; therefore, since the rate of change depends on the number of nuclei, radioactive decay is a process that is governed by exponential laws. If N (t) represents the number of nuclei at a given time t, then we can write :

$$N (t) = N_0 e^{-k t} \tag{1}$$

where  $N_0$  is the number of nuclei that existed at the beginning of the process (so in your activity last week,  $N_0$  was the number of M&Ms you counted at the very beginning), t is the time elapsed, and k is a constant. While all radioactive processes are governed by this relationship, the time scales on which they occur can vary tremendously. Some types of radioactive nuclei decay with a half life of nanoseconds (or less), and some nuclei, like U-238 half half lives of 4.5 billion years. Still, the number of nuclei of each type is described by equation (1) above!

Let's explore the behavior of exponential functions by looking at some graphs. Let's start by looking at the curve of the equation :

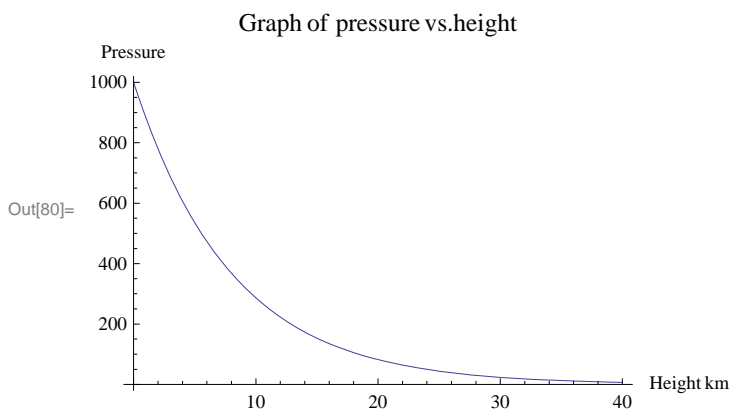
$$y = e^x$$



You can see that for negative values of  $x$ ,  $y$  is small but still positive, but that the value of  $y$  increases rapidly (in fact, exponentially) as  $x$  increases. Now, let's look at a common physical process governed by exponential laws. The pressure ( $P$ ) in the Earth's atmosphere decreases exponentially as you rise above the surface of the Earth. If we measure height in km above the surface of the Earth (and call the height variable " $z$ "), and measure pressure in a unit called millibars, then we can express the pressure at any height as :

$$P(z) = 1000 e^{-z/8}$$

we obtain :



Notice how the pressure decreases as height increases. What are the heights (in km) of the tallest mountains in the world. Use this graph to estimate the pressure at that altitude. Commercial jet liners cruise at elevations of about 35,000 feet. Convert this height to km and estimate the exterior pressure at cruising altitude. Can you understand why doors opening unexpectedly at this height is dangerous?

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## Cooling Curves :

Tonight we will do a series of measurements of the temperature of a cooling object (potatoes of different sizes). You all know that if we heat an object and then remove it from the oven or microwave, it will cool slowly. Tonight we will measure the rate at which the object cools, in other words, we will measure  $\Delta T/\Delta t$ , where T is the temperature of the object at a specific time t.

The rate at which objects cool is governed by an exponential law since the rate at which the object cools is proportional to the difference between the object's temperature and the surrounding temperature. To see what this means, suppose an object is at 100 C and the surrounding air is at 20 C; the rate of cooling is proportional to the difference of (100-20). As the object cools, its temperature decreases, and so does the difference in temperature between it and the outside environment. This means that the rate of cooling slows down: the cooler something gets, the longer it takes for the temperature to decrease by 1 degree.

We will investigate this behavior tonight. I will cut up some potatoes so that we will have potatoes of different sizes. Before we heat them up, measure the mass of each potato fragment; this is not the best way to describe their size, but it will help us draw some general conclusions.

Then we will heat each potato in the microwave for a few minutes, and then use thermometers to measure their temperatures. (If we heat the potato to a temp above the range of the thermometer, we will wait until the potato cools down to the range where its temp can be measured.)

Make sure you understand the divisions on each thermometer.

You will work in groups, although tonight group members will not work together. Each person will help take data for each potato, and you will combine each group member's data for your report (two weeks hence).

For each potato, measure the temperature of the potato at one minute intervals, being careful to record the temperature when  $t=0$ . Each person in the group should have one complete set of data for each potato. Share this data among all group members, then each group member should submit the data table for each potato (which should include the mass of the potato); additionally, plot these data for all potatoes on one graph of temperature vs. time (use some means to code the data, in other words, I should be able to identify the cooling curve for potato 1, for potato 2, etc.

Answer the following questions:

1) Do all the curves appear to be converging to the same temperature? What is your reasoning for your conclu-

sion.

- 2) If the answer to question 1 is yes, estimate the value of this temperature. What do you think the physical meaning of this temperature is.
- 3) For each potato, determine how long it took for the potato's temperature to decrease from its initial temperature by 10%; how long did it take to decrease the next 10%?; the next 10% (for each potato, you should have 5 to 7 decrements of 10%). What if any conclusions can you draw about the time it takes for the temp to decrease by 10% (does that time stay constant, increase, decrease or show no identifiable pattern of change).

Now, compare the "10% cooling times" for each potato. Is there any relationship between mass and 10% cooling time?