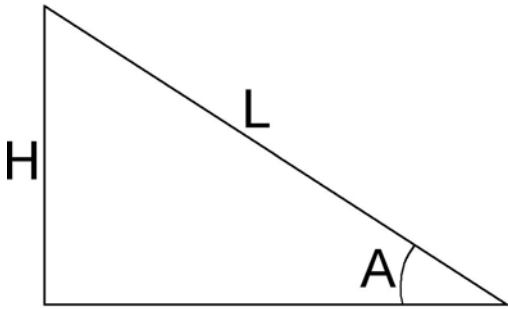


CONSIDERING THE H/L RATIO

We will use inclined planes in this course in our investigations of motion, work and machines. Thus, it is reasonable for us to consider in a bit more depth the nature of the H/L ratio we will encounter in these studies.

Let us consider a ramp of length L as shown below:



Here, H represents the height of the ramp, and A is the angle the ramp makes with respect to the ground. For our discussion here, it is important to note that H is the side **opposite** the angle A.

We know that making the ramp steeper corresponds to a greater value of A, and making the ramp less steep corresponds to a smaller value of A.

How does the value of the H/L ratio vary as we change the value of the ramp angle A? We can approach this in a couple of ways. First, let's consider **limiting cases**; this is a common technique used by mathematicians and physicists in studying a new problem.

Here, we consider the smallest and largest possible ramp angles, and ask what are the corresponding H/L ratios.

Let's start with the smallest angle possible; this of course would be a zero degree angle with the ramp lying on the ground. In this case, the height of the ramp is zero, and its length is still L, so the H/L ratio for a horizontal ramp (in other words, the H/L ratio for a zero degree angle) is 0.

Next, let's consider the steepest possible ramp. This case corresponds to a vertical ramp, or to a ramp making a 90 degree angle with respect to the ground. In this case, the height of the ramp is equal to its length, and of course its length is still L. This means that the H/L ratio for a vertical ramp is 1, since the height (H) now equals the length (L), so:

$$H/L = L/L = 1$$

These results tell us that the H/L ratio will indicate the angle of a ramp, and will have a numerical value ranging from zero to one.

When we investigate the motion of a cart on the inclined ramp, we will make measurements at many different angles, i.e., many different values of H/L.

How do different values of H/L actually equate to specific values of angles? The chart below shows the angles corresponding to different values of H/L:

H/L Ratio	Angle (degrees)
0.0	0
0.1	5.7
0.2	11.5
0.3	17.5
0.4	23.6
0.5	30
0.6	36.9
0.7	44.4
0.8	53.1
0.9	64.2
1.0	90

Notice that this table reproduces the data we deduced earlier; the H/L ratio is zero for a 0 degree angle; and the H/L ratio is 1 for a 90 degree angle.

The H/L ratio in a right triangle is so important that it is given a special name. The H/L ratio in a right triangle is called the **sine** of the angle. It is important to remember that sine of an angle is the ratio of height of the side **opposite the angle** to the length of the hypotenuse of the triangle. In other words, the **sine of angle A** in our picture above is merely our old friend, the ratio of H/L.

Interpolation—Calculating angles when your H/L ratio is not in the table

While the above table is very handy and shows the relationship between angle and H/L ratio, it probably does not give you an exact value of the angle corresponding to the H/L ratios you measured.

Suppose for example that the H/L ratio you measured was 0.15. The table lists the angles corresponding to H/L ratios of 0.1 and 0.2, but not to a ratio of 0.15. In this case, it is easy to argue that since 0.15 lies midway between two ratios in the table (namely, 0.1 and 0.2), it stands to reason that the angle corresponding to an H/L ratio of 0.15 has a value

midway between 5.7 and 11.5 degrees (which are the angles corresponding to H/L ratios of 0.1 and 0.2).

We argued just above that the angle corresponding to an H/L ratio of 0.15 lies midway between 11.5 and 5.7 degrees. The difference between the values of these two angles is $11.5 - 5.7$, or 5.8 degrees, so the angle we are looking for is $5.8/2$ degrees greater than 5.7 degrees. Thus, the angle corresponding to an H/L ratio of 0.15 is $5.7 + 2.9 = 8.6$ degrees.

How can we generalize this procedure to allow us to determine the angle of any H/L ratio? We use a procedure called **interpolation**.

Let's consider another example to see how the process works. Suppose you measured an H/L ratio of 0.68. We know that this H/L ratio is "bracketed" in the table by 0.6 and 0.7, so we recognize immediately that the required angle lies somewhere between 36.9 and 44.4 degrees, closer to the latter figure than the former (since 0.68 is closer to 0.7 than to 0.6).

Proceeding as before, we realize that 0.68 is 80% of the way between 0.6 and 0.7, so we expect that the appropriate angle is 80% of the way between 36.9 and 44.4 degrees.

How can we show this quantitatively? We compute the fraction:

$$(0.68 - 0.60)/(0.7 - 0.6) = 0.08/0.10 = 0.8$$

The numerator in this fraction (0.68-0.60) represents the distance between the measured angle and the value in the table just less than the measurement. The denominator (0.7-0.6) represents the difference between two table points.

Now we know that the angle corresponding to an H/L ratio of 0.68 lies 80% of the way between 36.9 and 44.4 degrees. So, we know that the difference between these two angles is $(44.4 - 36.9) = 7.5$ degrees. The angle we are looking for is then 0.8×7.5 degrees greater than 36.9 degrees, or has a value of 42.9 degrees.

Let's try one more: What is the angle when $H/L = 0.32$?

First, compute:

$$(0.32 - 0.30)/(0.4-0.3) = 0.02/0.1 = 0.2$$

Then, we know that the desired angle is greater than 17.5 degrees by an amount equal to:

$0.2 \times (23.6 - 17.5) = 0.2 \times 6.1 = 1.22$ degrees. Thus, the angle when $H/L = 0.32$ is equal to:

$$17.5 + 1.2 = 18.7 \text{ degrees}$$

