NTSC 395
SUPPLEMENTARY PROBLEM

A number of students have asked me to post some problems for practice. I will post solutions in a day or two. Please don' t hesitate to ask any questions. Have fun!

1. Make data tables and graph the following functions :

a) $y = x^2 - 3$

b) $y = 2x^3 - x^2 + 3$

c) $y = \sqrt{x^2 - 5}$

(are there any values of x for which y is not defined?, in other words, what is the range of y?)

d) $y = e^{-2x}$ (use your calculator to find the values of y)

e) $y = e^x$ (again, use your calculator and plot between $-3 < x < 3$)

2. Calculate the distance traveled by an object in the following cases :

a) An object starting from rest and accelerating at the rate of $2\ m/s^2$ for 5 seconds.

b) An object dropped from rest falling for a time of 3 s.

c) An object traveling at an initial constant speed of $5\ m/s$ being subjected to a force that accelerates it at the rate of $3\ m/s^2$. If the force is exerted at $t = 0$ s, how far will the object have traveled when $t = 3$ s.

3. Suppose an object is traveling at a speed of 20 m/s/s and is acted upon by a retarding force that causes it to accelerate at the rate of - 4 m/s. How long will it take the object to come to rest?

4. A converging lens has a focal length of 20 cm. If an object is placed 30 cm from the lens, where will the image be formed? If an object is placed 20 cm from the same lens, where will the image be formed. If an object is placed 10 cm from the same lens, where will the image be formed (review the sign conventions for a converging lens)

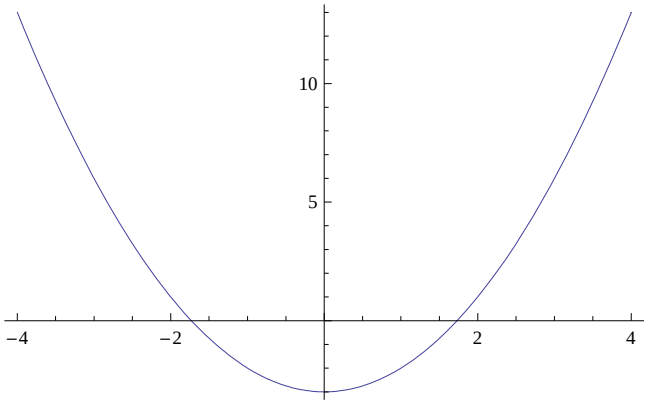
SOLUTIONS

1. $y = x^2 - 3$

x	y
0	-3
1	-2
-1	-2
2	1
-2	1
3	6
-3	6
4	13

In[1]:=
```
Plot[x^2 - 3, {x, -4, 4}]
```

Out[1]=

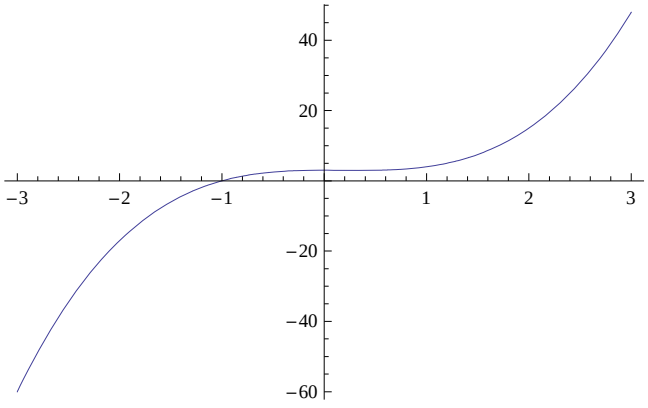


2. $y = 2x^3 - x^2 + 3$

x	y
0	3
1	4
-1	0
2	15
-2	-17
3	48
-3	-60

In[5]:=
```
Plot[2 x^3 - x^2 + 3, {x, -3, 3}]
```

Out[5]=

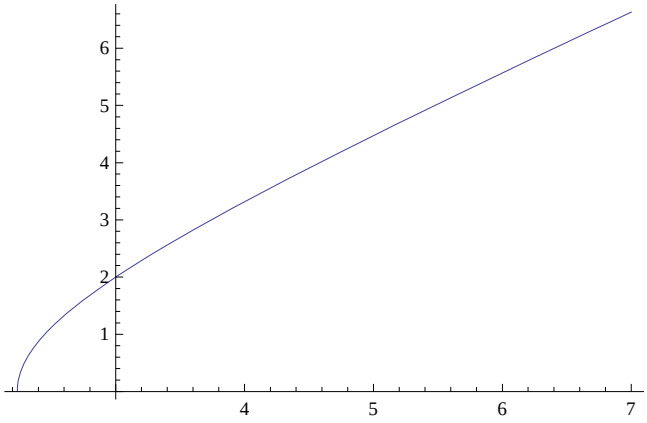


3. $y = \sqrt{x^2 - 5}$

x	y
3	2
4	3.32
5	4.47
6	5.57
7	6.63

In[8]:=
```
Plot[Sqrt[x^2 - 5], {x, Sqrt[5], 7}]
```

Out[8]=



For values of x less than the square root of 5, you will get a negative number in the radical; this will generate an imaginary number. So if we are graphing real numbers only, we must use values of x for which $\sqrt{x^2 - 5} > 0$.

4. $y = e^{-2x}$

x	y
-2	54.6
-1	7.39
0	1
1	0.13
2	0.02

In[9]:=
```
Plot[Exp[-2 x], {x, -2, 2}]
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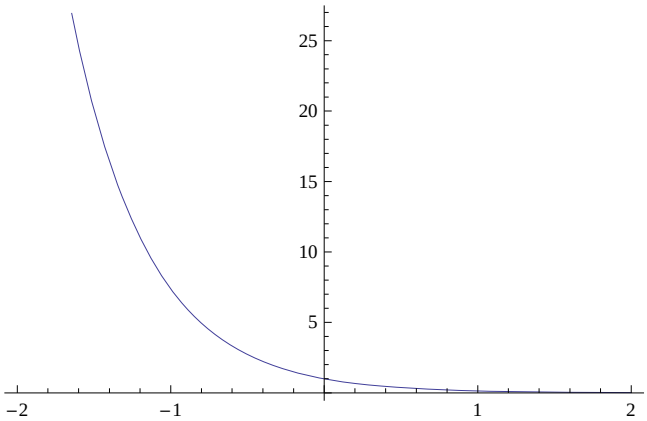
Out[9]=

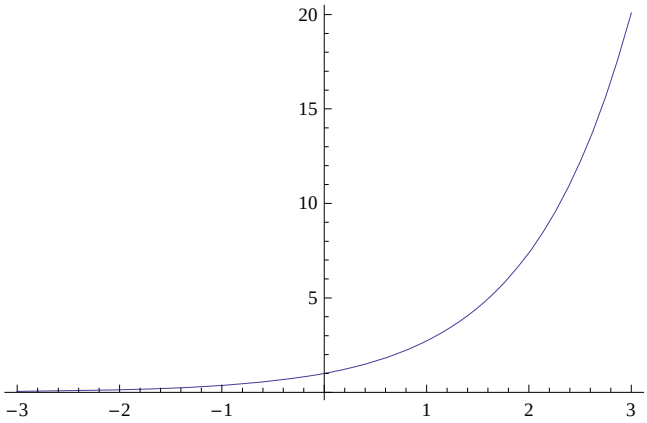


5. $y = e^{x}$

x	y
-2	0.13
-1	0.37
0	1
1	2.72
2	7.39

In[10]:=
```
Plot[Exp[x], {x, -3, 3}]
```

Out[10]=



Question 2 :

a) The appropriate equation to use in this case is :

$$\text{dist} = v_0 t + \frac{1}{2} a t^2$$

where v_0 is the initial speed, a is the acceleration and t is the time of travel. Here, we are told v_0=0, $a = 2\, m/s^2$and t=5s, therefore we have:

$$\text{dist} = 0 + \frac{1}{2} \left(2\,\text{m}/\text{s}^2\right)(5\,\text{s})^2 = 25\,\text{m}$$

b) We can use the same equation as in part a) realizing that here the initial speed is also zero, and that in this case the acceleration is the acceleration due to gravity which has a value of 9.8 m/s/s. So we have :

$$\text{dist} = v_0 t + \frac{1}{2} g t^2 = 0 + \frac{1}{2} 9.8\,\text{m}/\text{s}^2\,(3\,\text{s})^2 = 4.9\,\text{m}/\text{s}^2 \text{ x } 9\,\text{s}^2 = 44.1\,\text{m}$$

c) Use the same equation as in part a), with initial speed = 5 m/s, a = 3 m/s/s and t = 3 s :

$$\text{dist} = v_0 t + \frac{1}{2} a t^2 = 5\,\text{m/s} \times 3\,\text{s} + \frac{1}{2} 3\,\text{m/s/s} \times (3\,\text{s})^2 =$$
$$15\,\text{m} + 13.5\,\text{m} = 28.5\,\text{m}$$

Question #3 :

We use the definition of acceleration :

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{v_f - v_i}{t}$$

where v_f is the final speed and v_i is the initial speed. We are told the object comes to rest, so we know our final speed is zero. In essence, we are being asked to find the time it takes an object to come to rest from an initial speed of 20 m/s and slows down at the rate of 4 m/s/s. We have :

$$\text{acceleration} = -4\,\text{m/s/s} = \frac{v_f - v_i}{t} = \frac{0\,\text{m/s} - 20\,\text{m/s}}{t}$$

$$-4\,\text{m/s/s} = \frac{-20\,\text{m/s}}{t} \Rightarrow -4\,\text{m/s/s} \times t = -20\,\text{m/s}$$

$$t = \frac{-20\,\text{m/s}}{-4\,\text{m/s/s}} = 5\,\text{s}$$

Question 4

The relevant equation here is :

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

where o is the object distance, i is the image distance, and f is the focal length.

We are given three situations; where the object distance exceeds the focal length, where the object distance equals the focal length, and where the object distance is less than the focal length. We are also reminded to be mindful of sign conventions for converging lenses. The focal length of a converging lens always has a positive sign (the focal length is always a negative number for a diverging lens); the object distance is always positive, and the image distance is positive if the image is on the opposite side of the lens from the object (and the image distance is negative if the object and image are on the same side.)

In the first case, we have f = 20 cm and o = 30 cm, so we have :

$$\frac{1}{30} + \frac{1}{i} = \frac{1}{20}$$

$$\frac{1}{i} = \frac{1}{20} - \frac{1}{30} = \frac{3}{60} - \frac{2}{60} = \frac{1}{60}$$

Be careful to note that it is 1/i that equals 1/60 (do not make the common error of concluding that the image distance is 1/60). Since 1/i is 1/60, we have that the image distance is 60 cm; in other words, the image forms on the other side of the lens, 60 cm from the lens.

If we place the object at the focal length, we get :

$$\frac{1}{20} + \frac{1}{i} = \frac{1}{20} \Rightarrow \frac{1}{i} = \frac{1}{20} - \frac{1}{20} = 0$$

This means that 1/i is zero, which means that the image distance is infinitely far away. In other words, when you put the object at the focal length, no image forms (the physical meaning of an image forming at infinity.)

If the object distance is 10 cm :

$$\frac{1}{10} + \frac{1}{i} = \frac{1}{20} \Rightarrow \frac{1}{i} = \frac{1}{20} - \frac{1}{10} = \frac{1}{20} - \frac{2}{20} = \frac{-1}{20} \text{ and } i = -20 \text{ cm.}$$

This means that the image forms 20 cm from the lens, but on the same side as the object (so the image forms behind the object.)