

More on Optics

In our last class, you made a series of measurements of image and object distances using a converging lens. You should have found that as you decreased the object distance (i.e., moved the object closer to the lens) the image distance increased.

You then constructed a plot of (1/object distance) on the vertical axis vs. (1/image distance) on the horizontal axis.

Do these graphs appear to be linear or non-linear? Is the slope of the line (if it is a line positive or negative)?

Your data should present a linear relationship on this particular graph with a negative slope. Use the data you have to determine the slope of this line. We will consider the value of this slope in just a moment.

You are certainly familiar with linear relationships; you have studied these in math and science classes for years, and we reviewed them earlier this term. Remember that all linear relationships can be written in the form :

$$y = m x + b \quad (1)$$

Where y is the dependent variable (parameter plotted on the vertical axis), x is the independent variable (parameter plotted on the horizontal axis), m is the slope of the line (notice that if we write the equation in the form $y = m x + b$ then the coefficient of x is the slope of the line) and b is called the y intercept. It is the value of y when $x = 0$.

Now, let's consider how this equation for a line pertains to our current set of activities. Several of you asked why I had you use such seemingly unusual axes (1/distances) rather than just the distance. The reason is that I knew these particular axes would yield a linear relationship. By now you should realize that the slope of the line you obtain is minus one, which means that we can write the relationship between object distance and image distance as :

$$\frac{1}{\text{object distance}} = \frac{-1}{\text{image distance}} + b \quad (2)$$

where b is the y intercept and will be the value of (1/object distance) when (1/image distance) is zero. The negative sign on the right occurs because slope (m) is -1 in this case.

Now, let's think about the physical meaning of the value of b (the y - intercept). This will equal (1/image distance) when (1/object distance) is zero. But under what conditions will (1/object distance) be zero; in other words, what must be the value of the object distance for (1/object distance) to tend toward zero (or in the slang of science, when object distance goes to zero)?

In order for (1/object distance) to be zero, the object must be very, very far away (infinitely far away in fact if we want 1/object distance to be zero). So, when the object is infinitely far away, an image will still form; and the location of the image (when the object is far away) occurs at the **focal length** of the lens. This means that

if a lens has a focal length of 20 cm, and the object is placed very, very far away from the lens, then the image will be formed 20 cm from the lens.

We can put all this information together to derive an equation that relates object distance, image distance, and focal length. Let's go back to equation (2) and rewrite it by moving both distance terms to the left, we have :

$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{f} \quad (3)$$

Since we now know that the image distance equals the focal length (f) when the object distance is very, very large, we can see from eq. (3) that the y - intercept (given by the symbol b) is simply $1/\text{focal length}$, so that we can write our equation as :

$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{f} \quad (4)$$

For tonight and next week : Design an experiment to determine the focal length of your lens. You may use the same procedure we followed last week, but since you may have a different lens, take all new data tonight. Next week, turn in your data, include a graph of $(1/\text{image distance})$ vs. $(1/\text{object distance})$ (I realize this reverses the axes from last week; this is actually a better graph to use), calculate explicitly the slope of the line obtained from your data.

For each data point you have, calculate the focal length predicted by the data point (that means, calculate the focal length you obtain from that particular pair of object and image distances). Finally, when you have your graph complete, extrapolate the line you obtain to determine the value of $(1/\text{image distance})$ when the object distance becomes infinite (which means when $1/\text{object distance}$ goes to zero). This will give you a graphically predicted value of focal length; what focal length does your graph predict? How well does this graphical prediction match your values of focal length