

PLOTTING DATA AND INTERPRETING GRAPHS

Fundamentals of Graphing

One of the most important sets of skills in science and mathematics is the ability to construct graphs and to interpret the information they convey. In this write - up, we will investigate different graphs and their connection to the physics of motion.

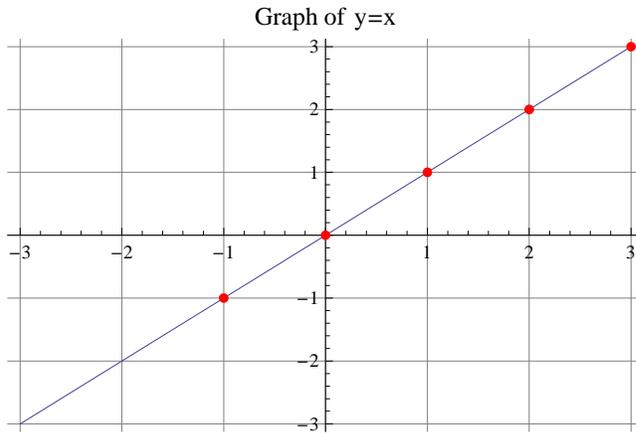
First, let's construct some simple graphs. We will start with one of the simplest graphs we know, the graph of the function $y = x$. In standard notation y is the dependent variable and is plotted on the vertical axis; x is the independent variable and is plotted on the horizontal axis.

Question for discussion : Think of some everyday examples of variables, such as the height and weight of a person. Which is the dependent variable and which is the independent variable? Try to think of some more examples of dependent and independent variables.

When we plot $y = x$ we are constructing a graphical representation of the relationship between the variables y and x ; in other words, we can look at this graph and understand how the variable represented by y relates to the variable represented by x . To construct the graph of an equation, we first create a table of values. So for this very simple case, we construct the table :

x	y
0	0
1	1
2	2
3	3
-1	-1

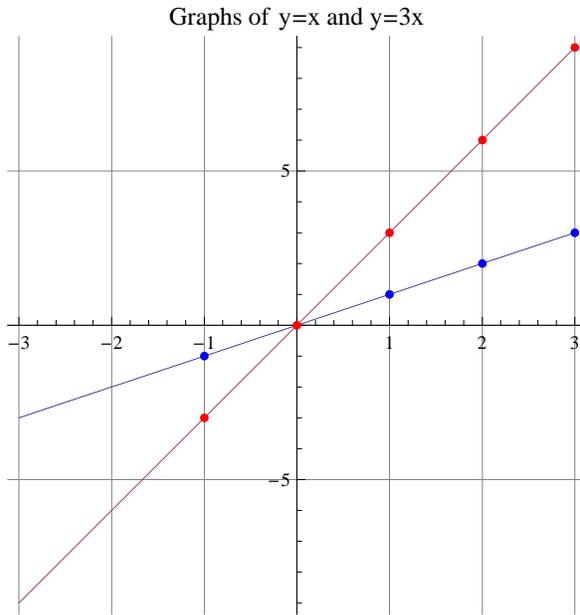
This table will provide us the information to plot each (x, y) point on our graph. Even though this is one of the the simplest equations we can graph, notice that it is good form to use some negative values of x (if negative values of x are meaningful for the equation) to ensure we understand the behavior of the graph . If we make use of the table above and plot those points on a graph (red dots below) and then draw the line that best connects the point, our graph should like something like :



Suppose now we want to construct a slightly more complicated plot, that of $y = 3x$. We should recognize that this graph is very similar to the one above, except that each value of y is three times its corresponding value of x . Our table of values is :

x	y
0	0
1	3
2	6
3	9
-1	-3

We can compare the behavior of the two graphs by superimposing them on one set of axes :



Where the data points are color coded (blue for the graph $y = x$ and red for the graph $y = 3x$).

Questions : How are the two graphs in the plot above similar? How are they different? What causes this difference in the two graphs?

Slope of a Line

You should be able to see that the graph of $y = 3x$ is steeper than the graph of $y = x$. You know from previous math courses that the steepness of a line is called its slope. The slope tells you the ratio of how the function is changing in the y direction compared to its change in the x direction. There are a number of ways of describing the slope mathematically, among the simplest is :

$$\text{slope} = \frac{\text{change in } y \text{ value}}{\text{change in } x \text{ value}}$$

We can use our tables of values above to determine the slope of the lines created by the equations $y = x$ and $y = 3x$. Let's start by finding the slope of the line $y = 3x$; the slope will be the ratio of the change in y value to change in x value. So as the function $y = 3x$ changes in x value from 1 to 2, its y value changes from 3 to 6 and we have :

$$\text{slope} = \frac{\text{change in } y \text{ value}}{\text{change in } x \text{ value}} = \frac{6 - 3}{2 - 1} = \frac{3}{1} = 3$$

and this is the value of the slope between the points $x = 1$ and $x = 3$. Since this is a straight line, we can see that the slope is the same at all points (this is a defining property of a straight line); let's verify this by comput-

ing the slope between the points $x = 0$ and $x = 3$. When $x = 0$, $y = 0$; when $x = 3$, $y = 9$, so :

$$\text{slope} = \frac{\text{change in y value}}{\text{change in x value}} = \frac{9 - 0}{3 - 0} = \frac{9}{3} = 3$$

and we should not be surprised to find the slope is still 3.

It should now be a simple task to compute the slope of the line $y = x$. What is the slope of this line ($y = x$)? Can you look at the equations for the two lines and determine what factor in those equations is responsible for determining the slope of the line?

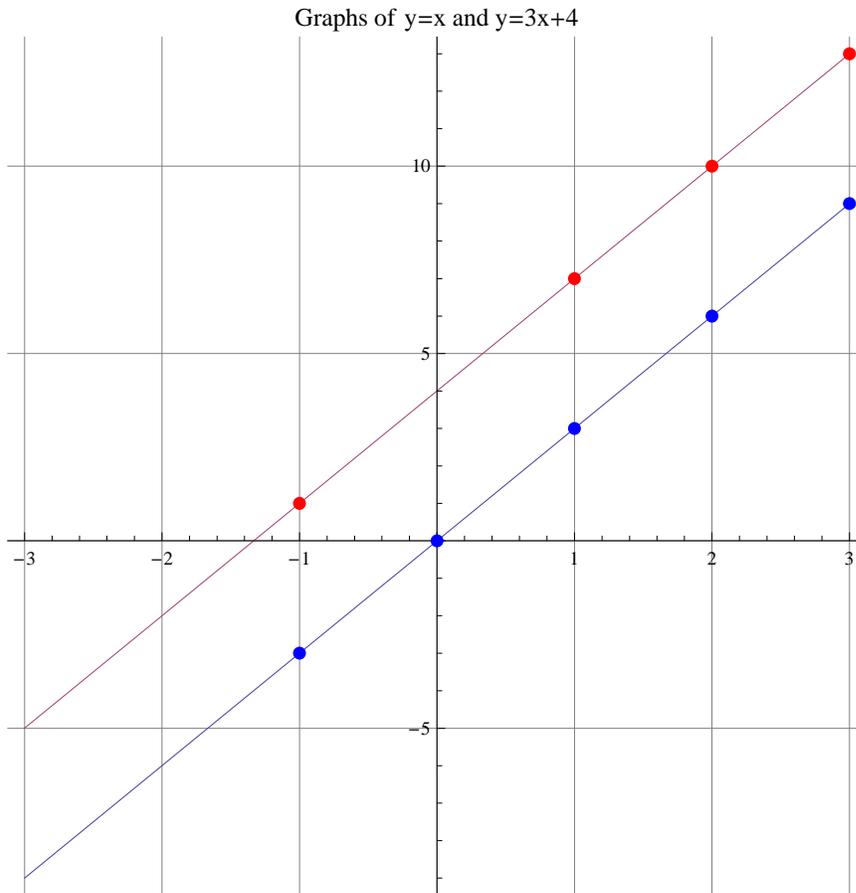
The y - intercept

Now, let's use a slightly more complicated function, $y = 3x + 4$. What do you think the slope of this line will be? (Make a prediction before we plot it). We can also construct this data table :

x	y
0	0
1	7
2	10
3	13
-1	1

Plot the graphs of $y = 3x$ and $y = 3x + 4$ on one set of axes one of the sheets of graph paper I have given you.

Now, after everyone has completed their graphs, let's see how your graph compares to :



Questions : How are these graphs similar? How are they different? Can you determine what are the factors in the equations that lead to the differences in these graphs?

The Equation of a Line

We are now in a position to determine a general equation that will describe all straight lines. This is a well known equation in math and is usually presented as :

$$y = mx + b \quad (1)$$

In this equation, y and x represent, as they often do, the dependent and independent variables; m represents the slope of the line, and b is the y - intercept, and is the y value of the straight line when it crosses the y axis.

So in our last equation above, we have $y = 3x + 4$. This means that we have a straight line of slope 3 and whose y intercept is 4.

When a line is written in the format of $y = mx + b$, we can immediately determine the slope by identifying the coefficient of the x term. We can also immediately determine the

y - intercept by identifying the constant term (i.e., the number added to or subtracted from the term containing "x".)

Questions : Consider the straight lines :

$$y = 2x + 4$$

$$y = -x$$

$$y = -2x - 3$$

What are the slopes and y - intercepts for each function? Construct a table of values for each equation and plot these three lines on one set of axes. (Put your name on your graph paper and submit these graphs and tables of values before you leave tonight).

Linear Equations

You may have heard the phrase "linear equations" or "linear functions" used in math and science. All of the equations we have plotted above are examples of linear equations; this is not surprising since we expect linear to be related a straight line. You will always have a linear (i.e., straight line relationship) between two variables when the equation that represents them involves only the first power of y and the first power of x. In other words, when both the exponents of y and x are 1, you will have a linear relationship between x and y and the graph displaying that relationship will be a straight line. However, if the equation that represents a relationship involves exponents that are different from one, then the graph of that relationship will be non-linear, meaning it will be curved in some way.

Make up a table of values for the equation $y = x^2 + 3$ and plot this equation. What does this graph look like? (Put your name on a graph and submit your table and graph tonight.)

Is it Linear or Non - linear?

Last week you measured the time of a car to go down a ramp of different values of (h/L); remember that the h/L ratio is related to the angle of the ramp. You used these data to determine the acceleration of the car down the ramp for each value of h/L. I also asked you to construct a graph of acceleration vs. (h/L) from your data. *Based on your data, do you think the relationship between acceleration and (h/L) is linear or non-linear. How do you know? A good metric would be to compute the slope of a straight line that connects successive data points; do you get the same slope (or nearly the same slope) for all pairs of data, or does the slope vary across the graph? Explain the reasoning behind your conclusion. (Copy down your data and submit your conclusions and data for next week. This should include your slope calculations.)* Your conclusions should be based on analyzing your own data.

Finally, you will determine if the distance an object travels down a ramp is a linear or non-linear function of time. In other words, if you plot the distance traveled down the ramp vs. the time of travel, would your graph be linear or non-linear?

Working in groups, construct an experiment (or series of experiments) that will allow you to measure the data you need to make this determination. You can use the same equipment from the last few weeks; ramps, carts, stopwatches and meter sticks. Once you have determined your procedure, carry out the observations needed.

For next week, each student should submit :

- a clear explanation of the procedure you used; you should write this up as if you were writing the lab instructions for a middle school class experiment*
- your data in a clear tabular form*
- your graph of distance vs. time*
- an explanation of your conclusion whether dist vs. time is a linear or non - linear relationship; your conclusion must be explicitly supported by your data.*

Each student should submit an individual report, but you should indicate who your lab partners were.