

USING THE INCLINED PLANE TO MEASURE SPEED AND ACCELERATION

Tonight and probably next week we will make measurements of the motion of an object to learn more about speed and acceleration.

The data you will take will be time of travel, the length of the trip, and also information that relating to the steepness of the incline of the ramp we use.

Measuring distance:

In order to compute such items as the average speed of the trip, we will need to measure the distance the moving object has traveled. In measuring distance, be sure that you measure the ramp length from where you actually begin the journey to the point that you call the end of the trip. Make sure that you release the object without giving it any forward push, this corresponds to the statement in physics that “the object starts from rest.”

Measuring Time:

We will also need to measure the time of the trip. It might take a few practice runs to be sure that you are starting your stopwatch at the moment (or as close to the moment of releasing the car as possible) and stopping it when the car reaches the end of its trip.

Measuring the Angle of the Incline

For now, we will measure the angle of the inclined plane by considering the ratio of the height of the ramp to the length of the ramp. To do this, you should measure the height from the table top to a specific point on the inclined plane. Then, measure the distance from that point to the bottom of the plane. You should be able to see that you can get the same height/length ratio by measuring at different points along the ramp, as long as you measure both height and length from the same point. I will provide you with information that will allow you to determine the angle of inclination, although it is equally fine to report results as the ratio of height/(ramp length), or what we will call the H/L ratio.

Calculating Speed and Acceleration

Using the carts, stopwatches and ramps, you will measure the time it takes a cart to travel down a ramp when the ramp is inclined at different angles.

Average Speed

The first calculation you will do will be to measure the average speed of an object moving down the plane. For this, you will simply take the total distance of travel and divide by the time it takes to reach the bottom.

You are very familiar with the notion of average speed. For instance, if you drive from Chicago to St. Louis, a distance of about 300 miles in 6 hours you would compute the average speed for the entire trip to be:

$$\begin{aligned}\text{Average speed} &= \text{total distance} / \text{total time} \\ \text{Average speed} &= 300 \text{ miles} / 6 \text{ hours} = 50 \text{ mi/hr}\end{aligned}$$

For at least five different angles of your inclined plane (ramp), you will measure the distance your car travels, the time it travels, and compute and report its average speed for the entire trip.

For each angle of the inclined plane, you will take three time measurements, and compute the average speed based on the average of the three times.

Instantaneous Speed and Acceleration

In the example of the trip from Chicago to St. Louis mentioned above, we recognize that the average speed of 50 mi/hr provides useful information about the large scale nature of the trip, but tells us nothing about the speed of the car at any particular moment. For instance, it is highly likely that the car was traveling much faster on the interstate in downstate Illinois (maybe 70 mi/hr or even 80 mi/hr), and was traveling much slower when stuck in Chicago traffic (maybe even being stationary for irritatingly long periods of time).

We use the concept of *instantaneous speed* to describe the motion of an object at an instant in time or at a particular place. We know that the average speed of the aforementioned car was 50 mi/hr, but the car's speedometer (which registers and displays instantaneous speed will vary throughout the trip.

Most people are familiar with the word *acceleration*. However, "acceleration" is used in a very specific way in physics. *Acceleration* refers to *any* change in the motion of an object. This change could be an increase in speed, a decrease in speed, or a change in direction. So, if a car travels around a circular track at a constant speed (of say for purposes of being specific 30 mi/hr), the car is in fact accelerating because the car is constantly changing direction. We will learn later that accelerations are closely related to *forces*; there is always at least one force acting on an object if that object is accelerating.

In our specific work with carts on a ramp, the only accelerations we will encounter will be accelerations due to the increasing speed of the object as it travels down the ramp. Let's see how we can calculate the acceleration of the car's motion. If we are dealing with accelerations that result from changes in speed, then the definition of acceleration is:

$$\text{Acceleration} = \text{change in speed} / \text{time}$$

This means that the acceleration of the car can be determined by figuring out how much its speed changed during the course of the trip. We can write this definition in an equivalent but slightly different way as:

$$\text{Acceleration} = (\text{final instantaneous speed} - \text{initial instantaneous speed})/\text{time}$$

The quantity in parentheses is just the change in speed, and it looks like a pretty straightforward task to compute the acceleration of the object.

But...all we know is the AVERAGE speed of the object, how can we find out the initial and final instantaneous speeds?

Well, if we are careful not to give the car a push at the beginning, we know the initial instantaneous speed is zero.

But what about the final instantaneous speed, in other words, how fast is the car going when it gets to the bottom of the ramp? How can you do this, since we have measured only average speeds?

Let's think for a moment what that average speed for the whole trip means. Let's think about an average that we are all familiar with, a test average. If you take two tests, you know your average on the two tests is:

$$\text{Avg test score} = (\text{Score on Test 1} + \text{Score on Test 2})/2$$

We can apply this same reasoning to the object's motion and recognize there is another way to think of the average speed for the whole trip:

$$\text{Average speed} = (\text{initial instantaneous speed} + \text{final instantaneous speed})/2$$

How does this help us get the final instantaneous speed? We know how we can calculate the average speed, since you will measure the time and distance of the cart down the ramp. And by being careful to start the object from rest, you know the initial instantaneous speed is zero. So, in this special case of the experiment as we have designed it, we know that:

$$\text{Average speed} = (0 + \text{final speed})/2$$

or:

$$\text{final instantaneous speed} = 2 * (\text{average speed})$$

This is an example of how we can deduce an important result without directly measuring it. If our measurements leading to the average speed for the whole trip are accurate and

we are careful not to give the cart a push at the beginning of the trip, then we can have confidence in the value of final instantaneous speed we deduce.

Now, we recall our definition of acceleration:

$$\text{Acceleration} = \text{change in speed/time of travel}$$

which becomes:

$$\text{Acceleration} = (\text{final instantaneous speed} - \text{initial instantaneous speed}) / \text{time of travel}$$

Since you measured the time of travel, and have just computed the final instantaneous speed, and know that the initial speed is zero, you can now compute the acceleration of the object down the ramp!

Procedure

Incline your ramp by resting one edge on a pile of books or other items. It is important that the ramp be stable and not slip during the course of your measurements. First, decide where you will start your cart's trip, and measure the length of the ramp from this starting point to the ending point.

Next, measure how high the track (the path the cart will take) is above the table at the point you will begin your motion. You will use these data to determine the H/L ratio.

For the angle of elevation you have set, use the stopwatches provided to measure the time it takes the car to travel down the incline. For each angle of elevation, make at least three measurements of the time. Record these measurements on the worksheet; later you will compute an average of the average speeds.

Once you have made three measurements of the time it takes for the cart to move down the ramp, change the angle of your ramp. Measure again the height of the track at the starting point and record this height. This will allow you to determine the H/L ratio for this angle.

Again, make three runs of your cart for this angle; recording each time on your worksheet.

Repeat this procedure for at least three more angles of your ramp. Record data and use this data to calculate the average average speed of the cart for that trip.

Calculations, Graph, and Questions

You will make the measurements described above with your group, and at home answer the questions below and turn them in to me next week. Turn in both your measured results on the worksheet and answers to the questions below. (If you don't complete all five angles this week, turn in what you have and we will finish up next week.)

1. For each angle of the ramp, compute the final instantaneous speed of the car. Show your work.
2. Knowing that the initial instantaneous speed was zero, calculate the acceleration of the cart for each elevation of the ramp. Show your work.
3. Use graph paper (or quadrille paper) and make a graph of your acceleration results; plot the acceleration of the cart vs. the H/L ratio. In other words, make a graph where the acceleration is plotted on the vertical axis and the H/L ratio on the horizontal axis.
4. Suppose we changed the design of the experiment slightly in the following way. Once you set the angle of your ramp, suppose you let the car start from midway up the ramp (about 1 meter on these ramps) and measured the time it took to reach the bottom. Then, with the ramp at the same angle, you started the car farther up the ramp (say twice as far up at around 2 meters) and measured the time it would take the cart to reach the bottom. Since the cart is traveling twice as far, will it take twice as long to reach the bottom? Explain if this is the case; if not, explain why it is not, and what is your prediction for the time to reach the bottom (faster than twice the previous time or slower than twice the previous time).