ELASTIC COLLISIONS

Your text omits many of the steps involved in determining the expressions for the final velocities of particles in an elastic collision. Elastic collisions are those in which kinetic energy is conserved, so that we can describe the motion of the particles in the system by using both the conservation of momentum and conservation of energy.

Let' s consider elastic collisions in 1 dimension. Object A is moving along the + x axis toward a stationary object B. The two objects collide elastically (no KE is lost), and both move along the x axis.

Our task is to find expressions for the final (post collision) velocities of A and B in terms of their masses and the original velocity of A. We begin by writing the laws of momentum and energy conservation :

$$m_A v_o = m_A v_1 + m_B v_2 \tag{1}$$

Where v_o is the initial velocity of A, v_1 is the velocity of A after collision, and v_2 is the velocity of B after collision. This notation differs from that used in your text, but I find that notation cumbersome, especially when I have to type.

The law of energy conservation becomes :

$$\frac{1}{2} m_A v_o^2 = \frac{1}{2} m_A v_1^2 + \frac{1}{2} m_B v_2^2 \tag{2}$$

We can cancel out the common factors of 1/2, yielding :

$$m_A v_o^2 = m_A v_1^2 + m_B v_2^2 \tag{3}$$

Now, as your book says, there is a bit of algebra to do. First, we rewrite equations (1) and (3) by grouping all the terms relating to object A on the left :

$$m_A (v_o - v_1) = m_B v_2 \tag{4}$$

$$m_A \left(v_o^2 - v_1^2\right) = m_B v_2^2 \tag{5}$$

Now, we can further rewrite eq. (5) by recognizing it is the difference of perfect squares. Recall from algebra that :

$$x^2 - y^2 = (x + y)(x - y)$$

So that equation (5) becomes :

$$m_A (v_o + v_1)(v_o - v_1) = m_B v_2^2 \tag{6}$$

Eq. (6) is the energy conservation in a slightly different form, and eq. (4) is the momenumt equation. We divide eq. (6) by eq. (4) :

$$\frac{m_A (v_o + v_1)(v_o - v_1)}{m_A (v_o - v_1)} = \frac{m_B v_2^2}{m_B v_2} \tag{7}$$

Cancelling common terms on each side :

$$v_o + v_1 = v_2 \tag{8}$$

Now, let' s go back to eq. (4) and substitute this expression for v_2 on the right hand side of eq. (4):

$$m_A (v_o - v_1) = m_B (v_o + v_1) \tag{9}$$

Collecting terms in v_1 we get:

$$v_1 (m_A + m_B) = v_o (m_A - m_B) \tag{10}$$

or :

$$v_1 = \frac{(m_A - m_B)}{(m_A + m_B)} v_o \tag{11}$$

and this is equivalent to equation 8.11 in your text. To find the final velocity of object B, we substitute the expression for v in eq. (11) into eq. (8) :

$$v_o + \frac{(m_A - m_B)}{(m_A + m_B)} v_o = v_2 \tag{12}$$

$$\frac{(m_A + m_B) v_o + (m_A - m_B) v_o}{m_A + m_B} = v_2 \tag{13}$$

Subtracting terms in eq. (13) yields the final result :

$$v_2 = \frac{2 m_A}{m_A + m_B} v_o \tag{14}$$

which is equivalent to eq. (8.12 in your text)

Now, suppose we have the special case where the two masses are equal. Equation (11) shows us that if the two masses are equal, the velocity of A after collision is zero. Eq. (14) shows that if the two masses are equal, the final velocity of B after collision is the same as the initial velocity of A.