1. Consider a car of mass m moving along a circular track of radius R. The track is banked with respect to the horizontal at an angle θ. The coefficient of friction between the tires and track is μ.The car is traveling just fast enough not to slide down the bank.

a) Draw a free body diagram showing all forces acting on the car. (10)

b) Derive an expression for the minimum velocity necessary for the car to keep from sliding down the bank. (20)

Solution : This was worked out in detail in Solutions #8, problem 1.

2. A bullet of mass m and velocity v is fired into a wooden block of mass M (the block is initially stationary and rests on a level frictionless surface). The bullet wedges completely into the block (this is a perfectly inelastic collision).

a) What is the speed of the bullet/block system after impact? (5)

Solution : We use the conservation of momentum :

$$m\,v = (m + M)\,V \Rightarrow V = \frac{m\,v}{m + M}$$

where V is the velocity of the combined mass after impact ..

b) What is the ratio of kinetic energy of the system after impact to the kinetic energy of the system before impact? (10)

Solution : We find the total KE before impact and the total KE after impact :

$$KE_i = \frac{1}{2} m\,v^2$$

$$KE_f = \frac{1}{2}(m + M)\,V^2 = \frac{1}{2}(m + M)\frac{(m\,v)^2}{(m + M)^2} = \frac{1}{2}\frac{(m\,v)^2}{m + M}$$

and the ratio of final to initial KE is :

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}\frac{(m\,v)^2}{m+M}}{\frac{1}{2} m\,v^2} = \frac{m}{m + M} = \frac{1}{1 + (M / m)}$$

c) The system then moves up a ramp inclined to the horizontal at an angle θ. The coefficient of sliding friction between the block and ramp is μ. The highest point the block reaches before friction slows it to rest is a distance L measured along the ramp. Find an expression for L in terms of m, M, v, θ, μ and g.

Solution : This part of the problem is a conservation of energy problem with a dissipative force. We equate the total KE at the bottom of the ramp to the total mechanical energy at the highest point, taking into account that there is negative work done by friction.

At the bottom, the initial KE is :

$$KE_i = \frac{1}{2}\frac{(m\,v)^2}{m+M}$$

At the top, the block/bullet system stops so has no KE, but has m g h of potential energy. We measure h from the level surface up; for a ramp inclined at angle θ, this height is equivalent to L sin θ where L is the length traveled along the incline. So the final PE is :

$$U_f = (m+M)\,g\,L\sin\theta$$

The work done by friction is :

$$W_f = -F_f\,L$$

and the force of friction is given by :

$$F_f = \mu\,N = \mu\,(m+M)\,g\cos\theta$$

since the component of gravity perpendicular to the incline is m g cos θ. Thus, our energy conservation for this situation becomes :

$$U_f + K_f = U_i + K_i + W_{other}$$

$$(m+M)\,g\,L\sin\theta + 0 = \frac{1}{2}\frac{(m\,v)^2}{m+M} - \mu\,(m+M)\,g\,L\cos\theta$$

Collecting terms in L :

$$(m+M)\,g\,L\,(\sin\theta + \mu\cos\theta) = \frac{1}{2}\frac{(m\,v)^2}{m+M}$$

or :

$$L = \frac{(m\,v)^2}{2\,(m+M)\,g\,(\sin\theta + \mu\cos\theta)}$$

3. A rotating device (say a turntable) has a radius R and angular velocity ω.

a) What is the moment of inertia, I, of the device if it has rotational kinetic energy K. (5)

Solution : From the definition of rotational kinetic energy :

$$KE_{rot} = \frac{1}{2}\,I\,\omega^2 \Rightarrow I = \frac{2\,KE_{rot}}{\omega^2}$$

b) If the device is in the shape of a solid disk, what is its mass? (5)

Solution : For the shape of a disk, we have that :

$$I = \frac{1}{2}\,M\,R^2 \Rightarrow \frac{1}{2}\,M\,R^2 = \frac{2\,KE_{rot}}{\omega^2} \Rightarrow M = \frac{2\,KE_{rot}}{(\omega\,R)^2}$$

4. For this problem, you will need to recall the Earth is rotating sphere of radius R and mass M. Consider a mass m on the equator of the Earth.

a) Draw a free body diagram of all forces acting on the mass. (5)

b) Find an expression for g in terms of G, M and R. (5)

Solution : We know that the weight of an object on the surface of the Earth can be alternately described via :

$$W = m g \quad \text{or}\, W = \frac{G\, m\, M}{R^2} \Rightarrow g = \frac{G\, M}{R^2}$$

c) If the Earth rotated rapidly enough, objects on the surface of the Earth would accelerate upward, away from the surface of the Earth. Derive an expression for v, the tangential velocity at which objects would begin losing contact with the Earth' s surface. Your answer should be in terms of G, M and R.(20)

Solution : The free body diagram from a) should show that the outward directed normal force and inward directed force of gravity combine to produce a centripetal force directed to the center of the Earth, or :

$$N - m g = \frac{-m v^2}{r}$$

Objects will lose contact with the Earth when N = 0 or when :

$$v^2 = r g$$

We can rewrite this as :

$$v^2 = R \cdot \frac{G\, M}{R^2} = \frac{G\, M}{R}$$

See if you can show that this requires a rotational period of 1.7 hrs!

(Hint : This question requires no knowledge of the material in Chapter 9)

5. A uniform chain of length L and total mass M hangs over the edge of a table. The coefficient of static friction between the chain (the portion of the chain on the table) and the table is μ. Represent the amount of the chain that is hanging over the table as x (so that the length on the table is L - x), what is the greatest length of chain that can overhang the table before the chain starts sliding? In other words, what is the greatest value of x before the chain begins to slide. Your answer should be in terms of μ and L. (30)

Solution : If the chain is uniform, then we can determine that the amount of mass in the overhanging portion is (x/L) M, and the amount of mass on the table is ((L - x)/L)·M. The forces acting on the chain are the weight of the overhanging portion and the frictional force between the chain and the table. If the chain is in equilibrium, these forces are balanced and we have :

$$\text{Weight of overhanging portion} = \left(\frac{x}{L}\right) M g$$

$$\text{force of friction between table and chain} = \mu \left(\frac{L - x}{L}\right) M g$$

Equating these yields :

$$\left(\frac{x}{L}\right) M g = \mu \left(\frac{L - x}{L}\right) M g \Rightarrow \frac{x}{L} = \frac{\mu}{1 + \mu}$$

FORMULAE AND RESULTS

$\Sigma F = m a$

$F_{cent} = \frac{m v^2}{r}$

$f = \mu N$

$F = \frac{G m_1 m_2}{r^2}$

$W = m g$

$a_{cent} = \frac{v^2}{r}$

$KE = \frac{1}{2} m v^2$

$U_{grav} = m g h$

$U_{elastic} = \frac{1}{2} k x^2$

$K_f + U_f = K_i + U_i + W_{other}$

$W = F s \cos\theta$

$F = - k x$

$p = m v$

$p_x = m v_x$

$p_y = m v_y$

$F \Delta t = \Delta (m v)$

$s = r \theta$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$

$\omega_f^2 = \omega_o^2 + 2 \alpha \theta$

$\theta(t) = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$

$I = \sum_i m_i r_i^2$

$I_{hoop} = M R^2 \quad I_{disk} = \frac{1}{2} M R^2 \quad I_{sphere} = \frac{2}{5} M R^2$

$I_{rod} = \frac{1}{3} M L^2$ (axis through one end)

$I_{rod} = \frac{1}{12} M L^2$ (axis through center)

$KE_{rot} = \frac{1}{2} I \omega^2$