PHYS 111
FIRST HOUR EXAMINATION
2016

This is a closed book, closed note exam. Do all your writing in your blue book(s) making sure your name is on each blue book you use. You may do question in any order as long as you indicate clearly which question you are answering. (It is a good strategy to read through the entire test and first work on the questions about which you are most confident.)

The use of calculators and other electronic devices is neither needed nor permitted on this test. Make sure all electronic devices are stored out of sight now.

All answers must be accompanied by complete work. No credit will be given for answers with no supporting work/explanations.

Please refer to the list of equations at the end of the exam.

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1. An large object moving through air at high speeds experiences air friction. The magnitude of this force is described by the equation :

$$\mathrm{F} = \rho^{\mathrm{a}}\, \mathrm{A}^{\mathrm{b}}\, \mathrm{v}^{\mathrm{c}}$$

where ρ is the mass density of the atmosphere, A is the cross sectional area of the object, and v is the speed of the object through the air. Use techniques of dimensional analysis to determine the values of the exponents a, b, and c, and thus determine the equation describing air friction. (15)
Some useful information:
- the units of force (newtons) are kg m s^{-2}
- the units of mass density are kg m^{-3}
- the units of area are m^2
- the units of speed are m s^{-1}

Solution : We begin by writing the equation in terms of the relevant units. On the left, we have units of force (kg m s^{-2}), and this must equal the units on the right:

$$\mathrm{kg\,m\,s^{-2}} = \left(\mathrm{kg\,m^{-3}}\right)^{\mathrm{a}} \left(\mathrm{m^2}\right)^{\mathrm{b}} \left(\mathrm{m\,s^{-1}}\right)^{\mathrm{c}}$$

We apply standard rules of exponents to the terms on the right to obtain:

$$\text{kg m s}^{-2} = \text{kg}^a\, \text{m}^{-3\,a}\, \text{m}^{2\,b}\, \text{m}^c\, \text{s}^{-c}$$

We see that there are three terms involving mass, so we combine them to get:

$$\text{kg m s}^{-2} = \text{kg}^a\, \text{m}^{-3\,a+2\,b+c}\, \text{s}^{-c}$$

We know that the units on the left must equal the units on the right. On the left, we have kg^1, and only one term involving kg on the right. This tells us that the value of a = 1. Similarly, we can equate terms involving s, and see that - 2 = - c or that c = 2.

Finally, we know that the combination of terms -3a + 2b + c must equal the exponent of m on the left, or that:

$$-3\,a + 2\,b + c = 1$$

We know that a = 1 and c = 2, therefore we have that:

$$-3 + 2\,b + 2 = 1 \Rightarrow b = 1$$

Our exponents are then: a = 1, b = 1, c = 2, and our final equation is:

$$F = \rho\, A\, v^2$$

(Actually, the complete equation involves some constants that don' t effect the units of any of the terms.)

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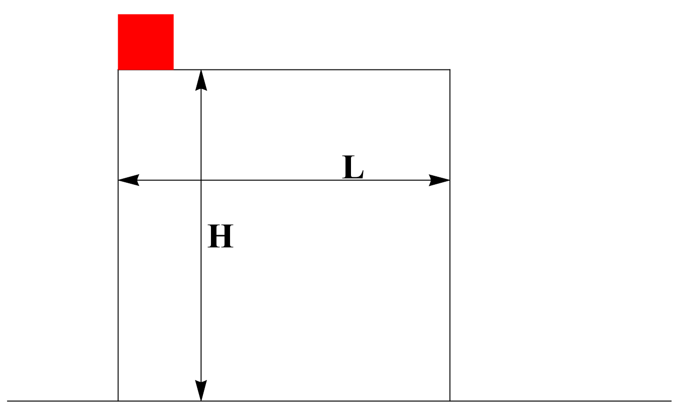
2. A box of mass m is initially at rest at the edge of a table of length L. The surface of the table is flat and frictionless, and lies a distance H above the ground. The box is pushed with a constant force until it reaches the opposite end of the table (a distance L away). (All answers for this question will be symbolic rather than numerical; this means that all answers should involve, as relevant, the parameters g, H, L, D, and m.). Assume air resistance is negligible. See diagram below :



a) Determine how long the box will be in the air . (Remember, you must show your work and not merely state an answer. It would be very helpful to define your coordinate system, i.e., are you choosing up or down to be positive?) (5)

Solution : We have solved this problem several times before on homework. Let' s adopt a coordinate system where down is positive (that means g will be positive). The initial motion of the box when it first leaves the table is all in the horizontal direction. Let' s call the initial horizontal speed v_o. The initial vertical speed is zero. To find the time of flight, we use the y(t) equation of motion:

$$y(t) = y_o + v_{oy}\, t + \frac{1}{2} g\, t^2 \text{ (remember, we are using down as positive so g is positive)}$$

Since down is positive, we can set our initial y value, $y_o = 0$, and the floor is at y = H. We want to find the time elapsed when y(t) = H, therefore we have:

$$H = 0 + 0 + \frac{1}{2} g\, t^2 \Rightarrow t = \sqrt{2\, H / g}$$

b) If the range of the box equals the height of the table (in other words, the box will travel a distance H from the edge of the table), find an expression for the speed of the box at the instant it left the table. (10)

Solution : Now, since there are no horizontal forces, we know there is no acceleration in the horizontal direction. Therefore, the horizontal motion of the box is constant. This means that the speed of the box when it leaves the table, v_o, will be the horizontal speed throughout the trip. Therefore, we can write the range of the box as:

$$\text{range} = v_o\, t = v_o \sqrt{2\, H / g}$$

But in this case we are given the additional information that the range is equal to the height, so we set range = H and get:

$$H = v_o \sqrt{2\, H / g}$$

Square both sides:

$$H^2 = v_o^2 (2\, H / g) \Rightarrow v_o^2 = \frac{g\, H}{2} \Rightarrow v_o = \sqrt{g\, H / 2}$$

c) Find an expression for the acceleration of the box as it moves across the table. (10)

Solution : We know the box starts from rest, and after traveling a distance L across the surface of the table achieves the speed we found in part b). Therefore, we can find the acceleration from the equation :

$$V_f^2 = v_o^2 + 2\, a\, x$$

where the initial speed here is zero, the final speed is $\sqrt{g\, H / 2}$, and the distance traveled is L. Therefore:

$$\frac{g\, H}{2} = 0 + 2\, a\, L \Rightarrow a = \frac{g\, H}{4\, L}$$

d) Find an expression for the force applied to the box. (5)

Solution : Since F = m a, we can write the force simply as F = m g H /(4 L)

e) What are the x and y components of the box' velocity at the instant it hits the ground? (10)

Solution : Since the horizontal component of motion does not change, the x component of velocity is the same as the initial x component of velocity, which we computed in part b) to be $\sqrt{g H / 2}$

The y velocity does change during the flight due to the force of gravity. The initial y velocity is zero; we can find the final y velocity from:

$$v_{fy}^2 = v_{oy}^2 + 2\,a\,y$$

Remember that we have chosen down to be positive, so the acceleration = +g, and the distance travled equals H. Since initial velocity is zero, we have :

$$v_{fy}^2 = 0 + 2\,g\,H \Rightarrow v_{fy} = \sqrt{2\,g\,H}$$

Alternatively, you could have started with :

$$v_y(t) = v_{oy} + a\,t \Rightarrow v_y(t) = 0 + g\,t = g\sqrt{2\,H/g} = \sqrt{2\,g\,H}$$

f) What is the magnitude of the final speed (the speed just as the box hits the ground)? (5)

Solution : The magnitude of the final speed is simply :

$$|\mathbf{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\frac{g\,H}{2} + 2\,g\,H} = \sqrt{\frac{5\,g\,H}{2}}$$

g) What is the angle of the final velocity vector with respect to the ground? (You may leave your answer in the form of $\tan\theta = \,.\,.$) (5)

Solution : And the angle between the final velocity vector and the ground is :

$$\tan\theta = \frac{v_{fy}}{v_{fx}} = \frac{\sqrt{2\,g\,H}}{\sqrt{\frac{g\,H}{2}}} = \sqrt{4} = 2 \Rightarrow \theta = 63.4^o$$

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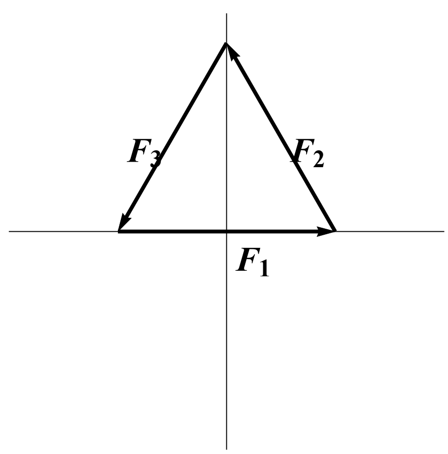
3. Three vectors, $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, and $\boldsymbol{F}_3$ form an equilateral triangle as shown below.



The length of each vector is one; since the triangle is equilateral, each interior angle is 60^o. Find the vector sum of $\boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$. Show your work and/or explain your logic if you do not do an explicit calculation. ($\cos 60^0 = 1/2$, $\sin 60^0 = \sqrt{3}/2$, $\tan 60^0 = \sqrt{3}$). (15)

Solution : There are two ways to approach this. You could write each vector in terms of $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$; find the components of each vector in the $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ directions, and sum those components to find the resultant vector. Or, you could recognize that the three vectors form a closed loop, meaning that the vector sum is zero. Imagine these three vectors were the three displacements of a trip; you are returnint to your starting point.

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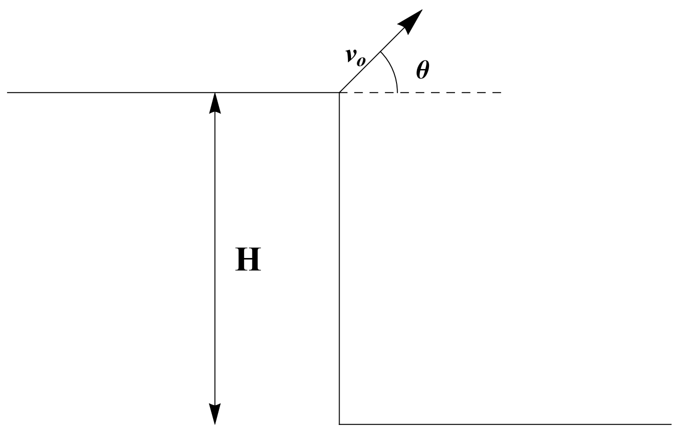
4. A projectile is launched from the edge of a cliff of height H. The launch angle is θ above the launch point and the initial velocity is v_o.



a) Use the equations of motion to find an expression for the time of flight in terms of g, H, θ and v_o. (10)

Solution : Let' s set up to be positive; then we have for our initial conditions that y_o = H, g is negative, and the initial y velocity is positive and is equal to $v_o \sin\theta$. It is convenient to set $x_o = 0$, $v_{ox} = v_o \cos\theta$, and since there are no horizontal forces, the acceleration in the x direction is zero. Then, our equations of motion become:

$$x(t) = x_o + v_{ox}\,t + \frac{1}{2} a_x t^2 = v_o \cos\theta\, t \tag{1}$$

$$y(t) = y_o + v_{oy}\,t + \frac{1}{2} a_y t^2 = H + v_o \sin\theta\, t - \frac{1}{2} g\, t^2 \tag{2}$$

We find the time of flight by determining the time when the projectile hits the ground, that is, when the projectile is at y = 0. This gives us:

$$0 = H + v_o \sin\theta\, t - \frac{1}{2} g\, t^2$$

This is a quadratic equation in t whose solutions are:

$$t = \frac{-v_o \sin\theta \pm \sqrt{(v_o \sin\theta)^2 + 2\,g\,H}}{-g} = \frac{v_o \sin\theta \pm \sqrt{(v_o \sin\theta)^2 + 2\,g\,H}}{g}$$

where all I have done in the last step is to multiply numerator and denominator by (-1). Since the radicand (stuff inside the square root) is larger that $v_o \sin\theta$, we have to take the positive branch of this solution to get a phyically meaningful, positive value of time.

b) Use the equations of motion to derive an expression for the height of the projectile as a function of its distance downrange; i.e., find an expression for y (x). (10)

Solution : For this part, we want to eliminate t from the equations of motion and find y in terms of x rather than y in terms of t. We begin by expressing t in terms of x. We do this by taking equation (1) from part a) and writing :

$$x = v_o \cos\theta\, t \Rightarrow t = \frac{x}{v_o \cos\theta}$$

We then substitute this expression for t into equation (2) wherever t occurs:

$$y(x) = y_o + v_o \sin\theta\left(\frac{x}{v_o \cos\theta}\right) - \frac{g}{2}\left(\frac{x}{v_o \cos\theta}\right)^2 = y_o + x\tan\theta - \frac{g\,x^2}{2\,v_0^2 \cos^2\theta}$$

LIST OF EQUATIONS AND RESULTS

$$v_{av} = \frac{\Delta x}{\Delta t} \tag{3}$$

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{4}$$

$$a_{av} = \frac{\Delta v}{\Delta t} \tag{5}$$

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \tag{6}$$

$$x(t) = x_o + v_{ox}\, t + \frac{1}{2} a_x t^2 \tag{7}$$

$$v(t) = v_o + at \tag{8}$$

$$v_f^2 = v_0^2 + 2\, a\, x \tag{9}$$

$$\mathbf{v}_{av} = \frac{\Delta \mathbf{r}}{\Delta t} \tag{10}$$

$$a_{cent} = \frac{v^2}{r} \tag{11}$$

$$W = m\, g \tag{12}$$

$$\Sigma \mathbf{F} = m\, \mathbf{a} \tag{13}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \tag{14}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \tag{15}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{16}$$

$$\sin^2\theta + \cos^2\theta = 1 \tag{17}$$

$$\sin 2\theta = 2 \sin\theta \cos\theta \tag{18}$$