PHYS 111
SECOND HOUR EXAM
2016

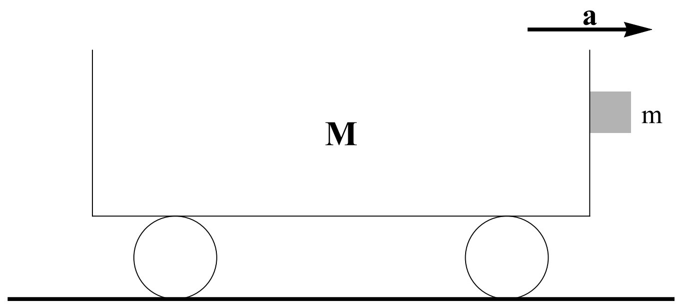
This is a closed book, closed note exam. Do all your writing in your blue book(s) and be sure to put your name on each blue book you use. All electronic devices (e.g., cell phones, calculators, laptops, tablets et al.) must be stored out of sight.

Read over the exam before starting; do questions in any order and it is a good idea to first finish the questions in which you are most confident. The value of each question is indicated by the numbers in parentheses.

All answers must be accompanied by complete work. No credit will be given for answers with no supporting work/explanations.

There is a list of equations and results at the end of this exam.

1. A cart of mass M moves along a frictionless horizontal floor with acceleration a. A box of mass m is positioned at the front of the cart as shown in the diagram below. If the coefficient of friction between the cart and the box is μ, what must be the value of the acceleration such that the box does not slide down the surface of the cart? (Your answer, of course, will include only symbols). (15)



Solution : The only horizontal force acting on m is the normal force due to contact with M. Since m is acceleration at a, we can write that N = m a. The box will not slide down if the force of static friction equals the weight of m. This condition can be written as:

$$f_s = m\,g$$

Since the force of static friction is given by $f_s = \mu_s$ N = μ_s m a, we have:

$$\mu_s\, m\, a = m\, g \Rightarrow a = \frac{g}{\mu_s}$$

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2. An open top railroad car coasts along frictionless, horizontal tracks at an initial constant speed V. When empty, the car has a mass M. Rain begins to fall and fills up the car. The rain falls vertically with respect to the Earth.

a) What will be the speed of the railroad car when an amount of rain of mass M/2 has fallen into the car? (10)

Solution : We apply conservation of momentum to this problem (see c below for why this is valid in this case). Our system will consist of the railroad car and the rain. Before the rain falls, the total horizontal momentum of the system is M V. Since there are no external forces acting on the system, we know the horizontal momentum will always be equal to M V. After an amount of rain equal to half the mass of the car has accumulated, we can write :

$$M\,V = \left(M + \frac{M}{2}\right) V_{\text{after}} \Rightarrow V_{\text{after}} = \frac{2}{3} V$$

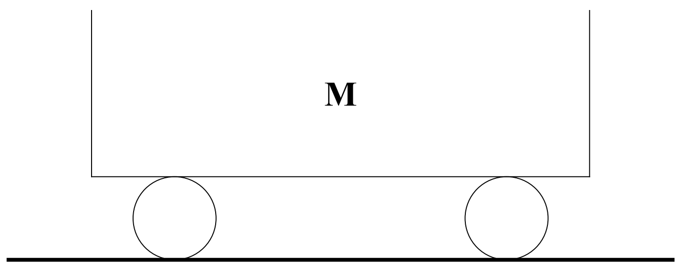
b) What is the ratio of initial to final kinetic energy (final kinetic energy refers to the KE when M/2 of rain is in the car)? (5)

Solution : We take the ratios of kinetic energies :

$$\frac{KE_i}{KE_f} = \frac{\frac{1}{2} M V^2}{\frac{1}{2} (3 M/2) (2 V/3)^2} = \frac{M V^2}{(3 M/2) \left(4 V^2 / 9\right)} = \frac{2}{3}$$

c) It was necessary for me to specify that the rain is falling vertically with respect to the Earth? What information does that give you that you need to solve this problem? (5)

Solution : Since momentum is a vector, you needed to know that there was no other momentum in the horizontal direction. If the rain had any horizontal component of motion, you would need to include that momentum in your calculations.



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3. Consider the diagram below. A bead of mass m is initially at rest at point A and slides along the surface of a frictionless sphere of radius R.

a) Draw a free body diagram of the forces acting on the bead at point B. (5)

b) Determine the speed of the bead at point B. (5)

Solution : This is worked out in detail on HW 9,
problem where we found that $v_B^2 = 2 g R (1 - \cos\theta)$

c) Consider the forces (and components of forces) acting in the radial direction at point B. Write Newton' s second law for the forces acting in the radial direction. (10)

Solution : Along the radial direction, we have the normal force acting out, and the component of weight acting toward O. These two forces combine to produce the centripetal force of the particle as it moves along the sphere. Newton' s second law becomes :

$$\Sigma F_{radial} = N - m g \cos\theta = \frac{-m v^2}{R}$$

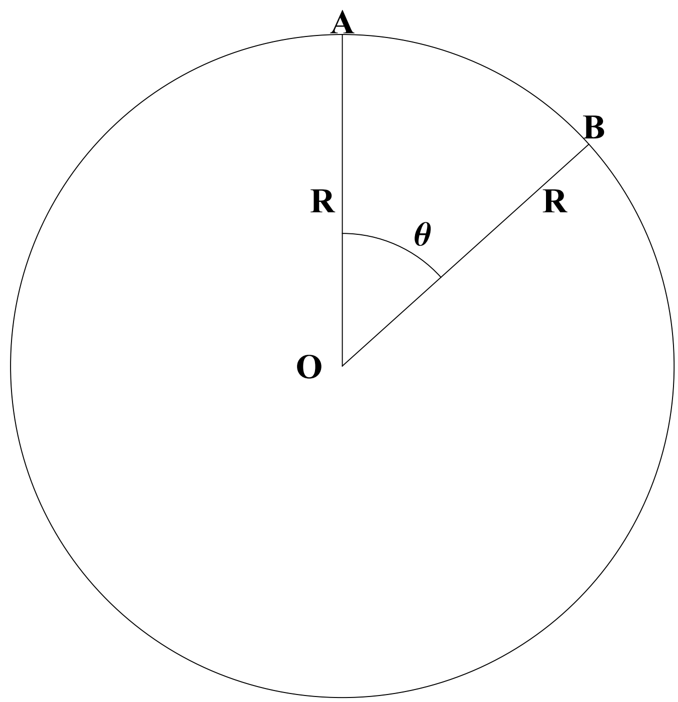
d) Find the angle θ at which the bead will leave the sphere. (10)

Solution : The bead leaves the sphere when N = 0 or when :

$$m g \cos\theta = \frac{m v^2}{R} \Rightarrow v^2 = R g \cos\theta$$

We can combine this result with the expression for speed derived in part a):

$$2 g R (1 - \cos\theta) = g R \cos\theta \Rightarrow \cos\theta = 2/3 \Rightarrow \theta = \cos^{-1}(2/3) = 48^o \text{ from the vertical.}$$



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4. Starting with the time independent equation of motion :

$$v_f^2 = v_o^2 + 2\,a\,s$$

derive the work energy theorem which equates work done to the change in kinetic energy. (15)

Solution : Let' s start with the equation above and multiply each term by 1 / 2 the mass, m :

$$\frac{1}{2}\,m\,v_f^2 = \frac{1}{2}\,m\,v_o^2 + m\,a\,s$$

We know that F = m a, and work = Fs, so the term m a s = Fs = work. Rearranging gives us:

$$\frac{1}{2}\,m\,v_f^2 - \frac{1}{2}\,m\,v_o^2 = \text{Work}$$

Finally we recognize that the terms on the left are just the final and initial kinetic energies, yielding the work energy theorem:

$$\Delta\,(\text{KE}) = \text{work}$$

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5. A particle of mass m is launched up an inclined plane of angle θ with an initial velocity of v. The coefficient of kinetic friction between the particle and the incline is μ_k.

After the mass has moved a distance L along the incline, it encounters a spring of spring constant k. Assuming the mass is still moving when it encounters the spring

a) Derive an expression for the speed of the particle when it encounters the spring. Your answer should be in terms of g, L, v, θ and μ_k. (10)

Solution : We approach this by conservation of energy. At the bottom, the mass has only kinetic energy, when it has traveled a distance L along the ramp, it has some (but less) kinetic energy, some gravitational potential energy, and has done work against friction.We write :

$$\frac{1}{2}\,\mathrm{m\,v^2} \;=\; \frac{1}{2}\,\mathrm{m\,v_L^2} \;+\; \mathrm{m\,g\,h} \;+\; \mu_\mathrm{k}\,\mathrm{m\,g}\cos\theta\,\mathrm{L}$$

where v_L is the speed when it has traveled a distance L along the ramp, h is the height above the starting point (after traveling a distance L along the ramp), and μ_k m g cos θ L is the work against friction. (The force of friction along the ramp is μ_k m g cos θ, so the work is jus the force times L). We can use the geometry of the situation to realize that sin θ = h/L or that h = L sin θ, so we can write:

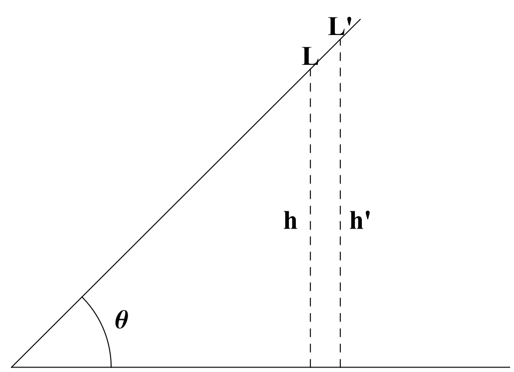
$$\frac{1}{2}\,\mathrm{m\,v^2} \;=\; \frac{1}{2}\,\mathrm{m\,v_L^2} \;+\; \mathrm{m\,g\,L}\sin\theta \;+\; \mu_\mathrm{k}\,\mathrm{m\,g}\cos\theta\,\mathrm{L}$$

Solving for v_L:

$$\mathrm{v_L} \;=\; \sqrt{\mathrm{v^2} - 2\,\mathrm{g\,L}\,(\sin\theta \;+\; \mu_\mathrm{k}\cos\theta)}$$

b) Derive an expression for how far the mass will compress the spring before it stops. (To simplify this part of the problem, assume there is no friction between the mass and incline in the region where the mass compresses the spring.) (10)

Solution : We apply conservation of energy again. At the moment the mass encounteres the spring, it has KE. As the mass compresses the spring, it also continues to move up on the ramp, so that there is an increase in gravitational potential energy. However, we simplified the problem so there is no friction and no energy lost to doing work against friction. Let' s consider the diagram below :



Let’s say the mass encounters the spring when it has traveled L units along the ramp (and is h units high), and has traveled L’ units when it fully compresses the spring (and is h’ units high). The mass compresses the spring by an amount (L’-L); in moving from L to L’, the mass gains m g (h’-h) in gravitational potential energy. Using the diagram we can see that this increase in GPE is simply m g(L’-L) sin θ.

So we can equate the mass' KE at L to its GPE and elastic energy at L':

$$\frac{1}{2}\,m\,v_L^2 \;=\; m\,g\,(L' - L)\sin\theta + \frac{1}{2}\,k\,(L' - L)^2$$

and this is a quadratic equation in (L'-L). We solve for L'-L:

$$L' - L \;=\; \frac{-\,m\,g\sin\theta \pm \sqrt{(m\,g\sin\theta)^2 + k\,m\,v_L^2}}{k}$$

LIST OF EQUATIONS

$\Sigma \mathbf{F} = m\,\mathbf{a}$

$F_{cent} = \dfrac{m v^2}{r}$

$v_f^2 = v_0^2 + 2 a s$

$f = \mu N$

$F = \dfrac{G m_1 m_2}{r^2}$

$W = m g$

$a_{cent} = \dfrac{v^2}{r}$

$KE = \dfrac{1}{2} m v^2$

$U_{grav} = m g h$

$U_{elastic} = \dfrac{1}{2} k x^2$

$K_f + U_f = K_i + U_i + W_{other}$

$W = F s \cos\theta$

$\Delta(KE) = W$

$\mathbf{F} = -k\,\mathbf{x}$

$\mathbf{p} = m\,\mathbf{v}$

$p_x = m v_x$

$p_y = m v_y$

$\mathbf{F}(\Delta t) = \Delta(m\,\mathbf{v})$

$P = \dfrac{W}{t}$