PHYS 111
OUTLINE OF SOLUTIONS
CHAPTER 10 (ODD PROBLEMS)

---

1. For each case, we can compute the magnitude of torque from :

$$\tau = F\, r \sin\theta$$

where F is the applied force, r is the distance to the pivot point (or rotation axis) and θ is the angle between the line of force and the radial line to the rotation axis. In each case, F = 10 N

a) $r = 4$ m, $\theta = 90^o$; $\tau = 40$ N m counterclockwise (ccw)

b) $r = 4$ m, $\theta = 120^o$; $\tau = 34.6$ N m, ccw

c) $r = 4$ m, $\theta = 30^o$; $r = 20$ N m, ccw

d) $r = 2$ m, $\theta = 60^o$; $\tau = 17.3$ clockwise (cw)

e) $r = 0$ so $\tau = 0$

f) $\theta = 0^o$ so $\tau = 0$

---

3. The 11.9 N force generates no torque since its line of action passes through the rotation axis. The 8.5 N force is tangential to the wheel. This means its line of action is perpendicular to the radius (so θ in this case is 90) and the torque generated is equal to 8.5 N · 0.35 m (2.97 N m and acts in the counterclockwise sense).

The 14.6 N force makes an angle of 50 degrees with respect to the tangent to the wheel, therefore the component of force tangent to the wheel is 14.6 cos 50, and the total torque generated is 14.6 N 0.35 m cos 50 (= 3.29 N m in the clockwise sense). Therefore, the total torque is 3.29 N m - 2.97 N m = 0.32 N m (clockwise)

---

5. The torques generated by these forces are:

- F_1 : $\tau_{F_1} = 18\text{ N} \cdot 0.09\text{ m} = 1.62$ N m clockwise
- F_2 : $\tau_{F_2} = 26\text{ N} \cdot 0.09\text{ m} = 2.34$ N m ccw
- F_3 : The force F_3 is at right angles to a line from the rotation axis to the corner of the square. Therefore,

the moment arm for this force is $\sqrt{(0.09\text{ m})^2 + (0.09\text{ m})^2} =$ 0.127 m and $\tau_{F_3} = 14\text{ N} \cdot 0.127\text{ m} = 1.78$ N m ccw.

The total torque is then 2.5 N m cw.

---

7. This problem requires us to equate torque, moment of inertia and angular acceleration via :

$$\tau = I\,\alpha$$

We have to use the information given to find the values of I and τ and from those compute α. The propeller can be modelled as a rod rotating through its center, so that :

$$I = \frac{1}{12} M L^2 = \frac{1}{12} \cdot 24\,\text{kg} \cdot (2.5\,\text{m})^2 = 12.5\,\text{kg}\,\text{m}^2$$

Since the force is applied perpendicularly to the blade, we know the moment arm is 1.15 m so that we have :

$$\tau = F\,r \sin\theta = 35\,\text{N} \cdot 1.15\,\text{m} \sin 90 = 40.25\,\text{N}\,\text{m}$$

Thus,

$$\alpha = \frac{\tau}{I} = \frac{40.25\,\text{N}\,\text{m}}{12.5\,\text{kg}\,\text{m}^2} = 3.22\,\text{rad} \big/ \text{s}^2$$

---

9. We are given information that allows us to compute the moment of inertia of the disk and its angular acceleration. This allows us to determine the torque needed to bring the rotating disk to rest. Since we know that $f = \mu N$, we use the given value of N to find μ. We begin :

$$I_{\text{disk}} = \frac{1}{2} M R^2 = \frac{1}{2} 50\,\text{kg}\,(0.26\,\text{m})^2 = 1.69\,\text{kg}\,\text{m}^2$$

(make sure you recognize that you are given the diameter and not the radius)

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega_f - \omega_o)}{\Delta t} = \frac{0 - 89\,\text{rad}/\text{s}}{7.5\,\text{s}} = -11.9 \frac{\text{rad}}{\text{s}^2}$$

Now, the torque is generated by the frictional force which acts at a distance R from the rotation axis, so we have :

$$\tau = I\alpha = f R \Rightarrow f = \frac{I\alpha}{R} = \frac{1.69\,\text{kg}\,\text{m}^2 \cdot \left(-11.9\,\text{rad}\,\text{s}^{-2}\right)}{0.26\,\text{m}} = -77\,\text{N}$$

Since $f = \mu N$, $\mu = \frac{f}{N} = \frac{77\,\text{N}}{162\,\text{N}} = 0.48$

---

11. This is very similar to other problems we have solved (see ex. 10.3 on p. 300). The difference is that we have to use the moment of inertia of a hollow cylinder. We write the linear and rotational versions of Newton' s second Law :

$$\Sigma F = T - m g = -m a \qquad (1)$$

$$\Sigma\tau = T R = I\alpha = I\left(\frac{a}{R}\right) \Rightarrow T = \frac{I a}{R_{\text{outer}}^2} \qquad (2)$$

where R is the distance from the center of the cylinder (i.e., the outer radius of 0.5 m) and the moment of inertia for a hollow cylinder is :

$$I = \frac{1}{2} M\left(R_{\text{inner}}^2 + R_{\text{outer}}^2\right) = \frac{1}{2}\,10\,\text{kg}\left((0.5\,\text{m})^2 + (0.3\,\text{m})^2\right) = 1.70\,\text{kg}\,\text{m}^2$$

Using this expression for T in eq. (1) :

$$\frac{I a}{R_{\text{outer}}^2} - m g = -m a \Rightarrow a = \frac{m g}{\left(I \big/ R_{\text{outer}}^2\right) + m}$$

Using values from the problem :

$$a = \frac{2\,\text{kg}\left(9.8\,\text{m}\,\text{s}^{-2}\right)}{\left(1.70\,\text{kg}\,\text{m}^2 \big/ (0.5\,\text{m})^2\right) + 2\,\text{kg}} = 2.23\,\text{m}\,\text{s}^{-2}$$

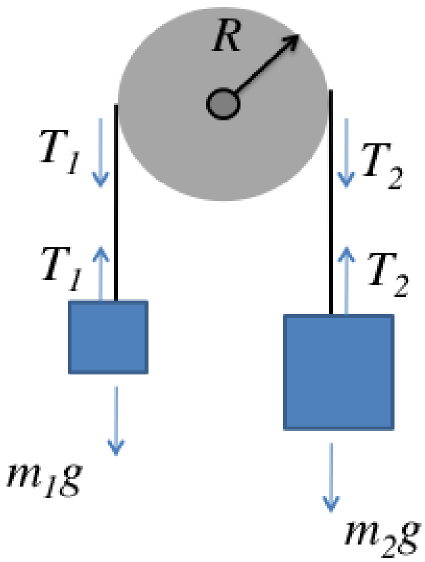
We find tension from eq. (1) :

$$T = m\,(g - a) = 2\,\text{kg}\left(9.81\,\text{m}\,\text{s}^{-2} - 2.23\,\text{m}\,\text{s}^{-2}\right) = 15.2\,\text{N}$$

and the angular acceleration is $\alpha = \frac{a}{R} = \frac{2.23\,\text{m}\,\text{s}^{-2}}{0.5\,\text{m}} = 4.46\,\text{rad}\,\text{s}^{-2}$

---

13. This problem really needs a diagram :



In this diagram, let' s call m_2 the counterweight and m_1 the elevator car. Unlike scenarios we encountered earlier in the semester, the pulley has mass M. The fact that the pulley has mass means that torque must be generated to turn it (unlike a massless pulley which requires no net force to cause it to rotate). If the pulley rotates clockwise (as the counterweight descends), there must be a net torque, and that means that $T_2 \neq T_1$. (If $T_1 = T_2$, there would be no net torque and the pulley could not rotate.). To analyze this problem, we write Newton's second law for the two weights, and the torque equation for the pulley:

Forces acting on m_1 : $T_1 - m_1\,g = +m_1\,a$ (the elevator care ascends) (3)

Forces acting on m_2 : $T_2 - m_2\,g = -m_2\,a$ (4)

Net torque on pulley : $T_2\,R - T_1\,R = I\,\alpha$ (5)

We use the expression for moment of inertia of a disk and the relationship $a = \alpha\,R$ and equation (5) becomes :

$$(T_2 - T_1)\,R = \frac{1}{2}\,M\,R^2\left(\frac{a}{R}\right) \Rightarrow T_2 - T_1 = \frac{1}{2}\,M\,a \qquad (6)$$

Now subtract equation (3) from equation (4) :

$$(T_2 - T_1) - m_2\, g + m_1\, g = -(m_2 + m_1)\, a \tag{7}$$

Regrouping and using the results of Equation (6) :

$$\frac{1}{2}\, M\, a + (m_2 + m_1)\, a = (m_2 - m_1)\, g \tag{8}$$

Solving for the mass of the counterweight, m_2:

$$m_2\, (g - a) = \frac{1}{2}\, M\, a + m_1\, (a + g) \tag{9}$$

We can find the acceleration of the system knowing the elevator starts from rest and moves upward 6.75 m in 3.00 s :

$$s = \frac{1}{2}\, a\, t^2 \Rightarrow a = \frac{2\, s}{t^2} = \frac{2\, (6.75\, m)}{(3\, s)^2} = 1.5\, \frac{m}{s^2}$$

We are given the weight of the elevator, and its mass is 22500 N/g so that :

$$m_1 = 2296\, kg$$

Substituting values :

$$m_2 = \frac{\frac{1}{2}\, M\, a + m_1\, (a + g)}{g - a} = \frac{\frac{1}{2}\, (875\, kg) \left(1.5\, m\, s^{-2}\right) + 2296\, kg \left(9.8\, m\, s^{-2} + 1.5\, m\, s^{-2}\right)}{\left(9.8\, m\, s^{-2} - 1.5\, m\, s^{-2}\right)} = 3204\, kg$$

Multiply by g to get its weight of 3.14×10^4 N

Substitute the masses and acceleration back into eqs. (3) and (4) to get the tensions in the string.

---

15. Was assigned for homework and is posted on the HW11 solutions.

---

17. This is similar to example 10.5 on pp. 302 - 303. The difference here is that the ball is rolling uphill rather than down.

The forces acting along the plane are the component of gravity down the plane (mg sin θ) and the frictional force (which acts down the plane as the ball rolls up the plane). Therefore, Newton' s second law along the plane is :

$$\Sigma F = -\, m\, g \sin\theta - f = m\, a \Rightarrow m\, a = -(m\, g \sin\theta + f)$$

The frictional force exerts a torque on the ball according to :

$$\tau = f\, R = I\, \alpha$$

Using what are by now well known relationships :

$$\tau = f\, R = \frac{1}{2}\, m\, R^2 \left(\frac{a}{R}\right) \Rightarrow f = \frac{1}{2}\, m\, a$$

Solving for a :

$$m\, a = -\, m\, g \sin\theta - \frac{1}{2}\, m\, a \Rightarrow a = -\frac{2}{3}\, g \sin\theta = -1.13\, m\, s^{-2}$$

If the initial velocity is 2.50 m/s, it will take a time :

$$t = \frac{\left(0 - 2.5\text{ m s}^{-1}\right)}{-1.13\text{ m s}^{-2}} = 2.20\text{ s}$$

---

23. a) Treating the Earth as a point particle (which is fine in considering its orbit around the sun), we have that L = m v r where m is the mass of the earth, v is the orbital velocity and r is the semi - major axis of the Earth' s orbit. This gives us :

$$L = 6.\times 10^{24}\text{ kg}\left(3\times 10^{4}\text{ m/s}\right)\left(1.5\times 10^{11}\text{ m}\right) = 2.70\times 10^{40}\text{ kg m}^2/\text{s}$$

b) $L = I\omega = \frac{2}{5} M R^2 \left(\frac{2\pi}{86\,400\text{ s}}\right)$

where M is the mass of the Earth, R is the radius of the Earth, and $2\pi/86400$ is the angular velocity. This yields :

$$L = \frac{2}{5}\cdot 6.\ 10^{24}\text{ kg}\cdot\left(6.38\times 10^{6}\text{ m}\right)^2\cdot\frac{2\pi}{86\,400\text{ s}} = 7\ \ 10^{33}\text{ kg m}^2/\text{s}$$

---

25. The angular momentum of a particle with respect to a rotation axis is L = m v r. At the start, v = 0 so the angular momentum is zero. Just before impact, the brick has a speed of 7 m / s (compute from $v_f^2 = v_o^2 + 2\text{ g s}$), so the angular momentum is :

$$L = 4\text{ kg}\cdot 7\text{ m/s}\cdot 1.6\text{ m} = 44.8\text{ kg m}^2/\text{s}$$

---

27. Once the cable snaps, the only force acting on the drawbridge is gravity. If the drawbridge is uniform, gravity will act at the center of mass which is at the midpoing of the bridge. The force is then 1500 N, the moment arm is 6 m · cos 60, so the torque is :

$$\tau = 1500\text{ N}\cdot 6\text{ m}\cos 60 = 45\,000\text{ N m.}$$

Torque is the time rate of change of angular momentum, so the answer is also 45000 N m.

---

29. I think the biggest challenge in solving this problem is getting through the verbiage. Quite simply, we need to apply the conservation of angular momentum to the skater, computing the moment of inertia with arms outstretched, and then with arms wrapped. Conservation of angular momentum tell us :

$$L_{\text{before}} = L_{\text{after}}$$

or :

$$I_{\text{before}}\,\omega_{\text{before}} = I_{\text{after}}\,\omega_{\text{after}}$$

where before refers to arms outstretched and after is when the arms are wrapped. When the arms are outstretched, we can think of them as a rod rotating around its center, so we have :

$$I_{\text{before}} = \frac{1}{12} M L^2 + I_{\text{rest of body}} = \frac{1}{12}\ 8\text{ kg}\,(1.8\text{ m})^2 + 0.4\text{ kg m}^2 = 2.56\text{ kg m}^2$$

When the arms are wrapped, we can think of them as a thin shelled cylinder (i.e., a hoop or ring) and the moment of inertia after is :

$$I_{\text{after}} = M R^2 + I_{\text{rest of body}} = 8\text{ kg}\,(0.25\text{ m})^2 + 0.4\text{ kg m}^2 = 0.9\text{ kg m}^2$$

Now, since we are told the initial angular velocity is 0.4 rev/s, we get :

$$\omega_{\text{after}} = \left(\frac{I_{\text{before}}}{I_{\text{after}}}\right)\omega_{\text{before}} = \left(\frac{2.56\ \text{kg m}^2}{0.9\ \text{kg m}^2}\right)(0.4\ \text{rev/s}) = 1.14\ \text{rev/s}$$

---

31. This is a conservation of angular momentum problem; it is similar to example 10.10 on pp. 310 - 311. Since there are no external torques acting on the system, angular momentum is conserved. This means that the angular momentum before collision equals the angular momentum after collision. Before collision, the angular momentum derives from the bird' s motion with respect to the pivot point; after collision, the angular momentum arises from both the bird' s motion and the rotation of the gate. So we have :

$$m\,v_o\,r = m\,v_a\,r + I\,\omega_{\text{after}}$$

Where m is the mass of the bird, v_o is its initial velocity (and let's call it positive), v_a is the velocity after collision (and will be negative), I is the moment of inertia of the gate (of mass M and length L), and ω is the angular velocity after collision. Refer to p. 279 for a listing of moments of inertia of various geometries. The relevant one for this problem is the plate rotating around an end:

$$I = \frac{1}{3} M L^2$$

Remembering that velocity is a vector (so the bird' s rebound velocity will have the opposite sign of its inbound velocity), we have :

$$\omega_{\text{after}} = \frac{m\,(v_o - v_a)\,L/2}{\frac{1}{3} M L^2} = \frac{1.1\ \text{kg}\,(5\ \text{m/s} - (-2\ \text{m/s}))/2}{\frac{1}{3}(4.5\ \text{kg} \cdot 1.5\ \text{m})} = 1.71\ \text{rad/s}$$

---

33. The key piece of physics here is that the child' s motion along the radius line does not effect the angular momentum of the system. Since the child' s motion is along a radial line, there is no external torque and there is no change in angular momentum. This becomes a simple conservation of angular momentum problem :

$$L_o = L_f \Rightarrow I_o\,\omega_o = I_f\,\omega_f$$

The initial angular velocity is 2 πrad/6 s = π/3 rad/s. We are told the initial moment of inertia (the child is at the rotation axis so contributes nothing to the initial value of I). After the child moves 2 m away from the axis, the new moment of inertia is :

$$I_f = 1200\ \text{kg m}^2 + m_{\text{child}}\,r^2 = 1200\ \text{kg m}^2 + 40\ \text{kg}\,(2\ \text{m})^2 = 1360\ \text{kg m}^2$$

We have :

$$\omega_f = \left(\frac{I_o}{I_f}\right)\omega_o = \left(\frac{1200\ \text{kg m}^2}{1360\ \text{kg m}^2}\right)(\pi/3\ \text{rad/s}) = 0.924\ \text{rad/s}$$

---

63. We can solve this problem using the results of problem 13 above. Refer to the equations derived in that problem to see that :

Forces acting on m_1 : $T_1 - m_1 g = +m_1 a$ (m_1 is the lighter mass)

Forces acting on m_2 : $T_2 - m_2 g = - m_2 a$

$$\text{Torque acting on pulley : } (T_2 - T_1) R = I\left(\frac{a}{R}\right) \Rightarrow (T_2 - T_1) = \frac{I}{R^2} a \quad (10)$$

Follwing the procedure of problem 13, and subtracting the first two equations :

$$(T_2 - T_1) + (m_2 + m_1) a = (m_2 - m_1) g \quad (11)$$

Substitute eq. (10) into eq. (11) :

$$\left(\frac{I}{R^2} + m_2 + m_1\right) a = (m_2 - m_1) g$$

$$a = \frac{(m_2 - m_1) g}{\left(\frac{I}{R^2} + m_2 + m_1\right)} = \frac{(4\text{ kg} - 2\text{ kg})\, 9.8\text{ m s}^{-2}}{\left(\frac{0.3\text{ kg m}^2}{(0.12\text{ m})^2} + 4\text{ kg} + 2\text{ kg}\right)} = 0.730\text{ m s}^{-2}$$

Find the angular acceleration by dividing a by R (the answer in the book is correct). Use the tension equations with the derived value for acceleration to find the tensions. Again, the tension is not the same in the cable since there must be a net torque to rotate the massive pulley.

---

69. I think this is an interesting problem, although I probably can' t ask this on the final since it deals with the two conditions of equilibrium. The wire exerts a clockwise torque on the cylinder; in order for the cylinder to remain in equilibrium, there must be a force generating a ccw torque. As we discussed in class, neither gravity nor the normal force can provide this torque, since they act through the center of the cylinder and thus have no moment arm (and therefore no torque). There must be a frictional force exerting the opposing torque. Now, we have to be careful as to the direction of the frictional force. Looking at the figure (10.79), the friction acts up the plane. This is necessary for friction to generate a counterclockwise torque. The frictional force must be equal in magnitude to T so that they have equal but opposite torques.

We can determine the magnitude of the force by resolving forces acting down the plane. T and f are of equal magnitude and act up the plane. They must be balanced by the component of gravity down the plane, so we have :

$$\Sigma F_{\text{down plane}} = T + f - m g \sin\theta = 0$$

$$\text{Since } T = f,\ 2T = 2f = m g \sin\theta \Rightarrow f = T = m g \sin\theta / 2$$

---

71. This is a classic problem. The forces acting on the wheel are gravity and F. If F acts at the center (as shown in the diagram), it generates a torque around the corner edge of F (R - h) (because R - h is the moment arm of the force F). Gravity exerts a torque in the opposite direction. We have to use the Pythagorean Theorem to determine the moment arm due to the weight. The weight acts from the center of the wheel down; its moment arm is the base of the triangle whose hypotenuse is the radius line from the center of the wheel to the edge of the curb and whose altitude is R - h. By the Pythagorean theorem, the length of this side, and therefore the moment arm is :

$$\text{moment arm} = \sqrt{R^2 - (R - h)^2} = \sqrt{2 h R - h^2} \Rightarrow$$

$$\tau = m\,g\sqrt{2\,h\,R - h^2}$$

The wheel begins to rotate over the curb when the torque due to F just exceeds the torque due to the wheel' s weight or when :

$$F\,(R-h) = m\,g\sqrt{2\,h\,R - h^2} \Rightarrow F = \frac{\sqrt{2\,h\,R-h^2}}{R-h}$$

b) If F is exerted at the top of the wheel, the only thing that changes is the moment arm of F which now begins (2 R - h). The torque due to the weight is unchanged, so our torque equilibrium condition is :

$$F\,(2\,R-h) = m\,g\sqrt{2\,h\,R-h^2} \Rightarrow F = \frac{\sqrt{2\,h\,R-h^2}}{2\,R-h}$$

Your book' s answer has a typo (the "v" should be the superscript "2").

---

73. This is simply a conservation of angular momentum problem. The star is spherical before and after collapse, and no mass is gained or lost in the process. Therefore, conservation of angular momentum tells us :

$$\omega_f = \omega_o\left(\frac{I_o}{I_f}\right) = \omega_o\left(\frac{R_o^2}{R_f^2}\right) = \omega_o\left(\frac{7\times10^5}{16}\right)^2 = 1.9\times10^9\,\omega_o = 1.9\times10^9\,(2\,\pi/30\text{ days})$$

Since there are 86400 sec/day, the new angular velocity is 4637 rad/s