CHAPTER 9 PROBLEMS
OUTLINE OF SOLUTIONS

1. We use the relationship s = r θ ; s is arc length, r is radius, θ is angle subtended.

$$\theta = \frac{s}{r} = \frac{0.75\text{ m}}{2.5\text{ m}} = 0.3\text{ radians} = 17.2\text{ degrees.}$$

3. The second hand completes 2 π radians in 60 s, so ω = 2 π rad/60 s = 0.10 rad/s. The minute hand completes 2 π radians in 3600 seconds, so ω = 2 π rad/3600 s = 0.0017 rad/s. The hour hand completes 2 π radians in 12 hours (assuming a 12 hour clock as is typical in the United States); so its angular velocity is 1/12 that of the minute hand.

5. $\omega = 2\pi / 2.25\text{ s} = 2.779\text{ rad/s}$. The period is stated; it is 2.25 s.

7. Distance to the moon is 384, 000 km. v = r ω so the linear velocity across the moon is :

$$v = 3.84 \times 10^8\text{ m} \cdot 1.5\ 10^{-3}\text{ rad/s} =$$
$$5.76 \times 10^5\text{ m/s (make sure to convert distance to moon to meters).}$$

We can find the half angle of divergence from :

$$\tan\theta = 3\text{ km} / 384,000\text{ km} = 7.81 \times 10^{-6}$$

Remember that for small angles, and this certainly qualifies, tan $\theta \approx \theta$ (measured in radians), so the half angle is 7.81 10^{-6} rad, and the full angle is twice this.

9. a) $1900\text{ rev/min} = 1900\text{ rev/min} * (2\pi\text{ rad/rev})(1\text{ min}/60\text{ s}) = 198.9\text{ rad/s}$

b) $\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta}{\omega} = \frac{35^o * 2\pi\text{ rad}/360^o}{198.9\text{ rad/s}} = 0.0031\text{ s}$

c) $18\text{ rad/s} = 18\text{ rad/s} * 1\text{ rev}/2\pi\text{ rad} * 60\text{ s/min} = 172\text{ rev/min}$

d) The period is the time to make one revolution
so if the propeller has an angular velocity of 199 rad / s,
it will complete 1 rev (2 π rad) in $2\pi\text{ rad}/199\text{ rad/s} = 0.032\text{ s}$

11. $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{78\text{ rev/min} * 2\pi\text{ rad/rev} * 1\text{ min}/60\text{ s}}{3.5\text{ s}} = 2.33\text{ rad}/\text{s}^2$

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 = 0 + 0 + 0.5 * 2.33\text{ rad}/\text{s}^2 * (3.5\text{ s})^2 = 14.3\text{ rad} = 818^o$

13. $570\text{ rpm} = 570\text{ rev/min} * 2\pi\text{ rad/rev} * 1\text{ min}/60\text{ s} = 60\text{ rad/s}$

$1600\text{ rev/min} = 167\text{ rad/s}$

$\alpha = \frac{\Delta\omega}{\Delta t} = (167\text{ rad/s} - 60\text{ rad/s})/(133\text{ min} * 60\text{ s/min}) = 0.0135\text{ rad}/\text{s}^2$

15. $\alpha = \frac{\Delta\omega}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\omega}{\alpha} = (8\text{ rad/s} - 0)/0.64\text{ rad}/\text{s}^2 = 12.5\text{ s}$

$$\theta = \theta_o + \omega_o + 0.5\,\alpha\,t^2 = 0 + 0 + 0.5 * 0.64\,\frac{\text{rad}}{\text{s}^2} * (12.5\text{ s})^2 = 50\text{ rad} = 7.96\text{ revs}$$

(For the remainder of the problems, I will not write out explicitly converstion from rpm to rad/s)

17. We find the angular acceleration :

$$\theta = \omega_o\,t + \frac{1}{2}\,\alpha\,t^2$$

$$\theta = 200\text{ rev} = 1256\text{ rad};\ \omega_o = 500\text{ rpm} = 52.3\text{ rad/s};\ t = 30\text{ s}$$

$$1256\text{ rad} = 52.3\text{ rad/s} * 30\text{ s} + 0.5 * \alpha\,(30\text{ s})^2$$

$$-314\text{ rad} = 0.5 * \alpha\left(900\text{ s}^2\right) \Rightarrow \alpha = \frac{-314\text{ rad}}{0.5 * 900\text{ s}^2} = -0.7\,\frac{\text{rad}}{\text{s}^2}$$

Knowing the angular acceleration, we know

$$\frac{\Delta\omega}{\Delta t} = \alpha \Rightarrow \omega_f - \omega_o = \alpha\,t$$

$$\omega_f = \omega_o + \alpha\,t = 52.3\text{ rad/s} - 0.7\text{ rad}\big/\text{s}^2 * 30\text{ s} = 31.3\text{ rad/s}$$

At this angular acceleration,
it would take $\Delta t = \Delta\omega/\alpha = -52.3\text{ rad/s}/-0.7\text{ rad}\big/\text{s}^2 = 74.7\text{ s}$ to stop

In this time, the flywheel will spin through the angle :

$$\theta = \omega_o\,t + 0.5 * \alpha\,t^2 = 52.3\text{ rad/s} * 74.7\text{ s} - 0.5 * 0.7\text{ rad}\big/\text{s}^2 * (74.7\text{ s})^2 = 1954\text{ rad}$$

19. We can use

$$\theta = \omega_o\,t + 0.5 * \alpha\,t^2$$

$$\omega_o = \frac{\left(\theta - 0.5\,\alpha\,t^2\right)}{t} = \frac{60\text{ rad} - 0.5 * 2.25\text{ rad}\big/\text{s}^2 * 16\text{ s}^2}{4\text{ s}} = 10.5\text{ rad/s}$$

21. Ugh unit conversions. We will use $v = r\,\omega$, but first :

$$v = 63\text{ mi/h} = 63\text{ mi/h}\,(5280\text{ ft/mi} * 1\text{ m}/3.28\text{ ft}) * 1\text{ h}/3600\text{ s} = 28.2\text{ m/s}$$

$$r = 12\text{ in} = 30\text{ cm} = 0.3\text{ m}$$

$$\omega = \frac{v}{r} = \frac{28.2\text{ m/s}}{0.3\text{ m}} = 93.9\text{ rad/s}$$

23. $a_{rad} = (6\text{ rad/s})^2 * 0.5\text{ m} = 18\text{ m}\big/\text{s}^2$

$$v = \omega\,r = 6\text{ rad/s} * 0.5\text{ m} = 3\text{ m/s}$$

$$a_{rad} = \frac{v^2}{r} = \frac{(3\text{ m/s})^2}{0.5\text{ m}} = 18\text{ m}\big/\text{s}^2$$

25. If the chain doesn' t slip, the chain must have the same linear velocity throughout. Therefore,

$$v_8 = v_3 \Rightarrow 8\,\omega_8 = 3\,\omega_3 \Rightarrow \omega_8 = \frac{3}{8}\,\omega_3$$

where ω_8 is the angular velocity of the 8 inch wheel,
and ω_3 is the angular velocity of the 3 inch wheel. If the three inch wheel turns at 75 rpm,
the 8 inch wheel turns at 3 / 8 of this, or 28 rpm

27. The radius of the drill bit is 6.35 mm = 0.00635 m. The angular velocity is 130.8 rad/s. The maximum linear speed is then

$$v = \omega r = 130.8 \text{ rad/s} * 0.00635 \text{ m} = 0.831 \text{ m/s}$$

$$a_{rad} = \frac{v^2}{r} = \frac{(0.831 \text{ m/s})^2}{0.00635 \text{ m}} = 109 \text{ m} \big/ s^2$$

29. $a_{rad} = \omega^2 r$

so the ratio of accelerations is :

$$\frac{\omega_{max}^2}{\omega_{min}^2} = (640/423)^2 = 2.29$$

(since the radius is the same for the both, it does not effect the ratio).

Tangential speed is $v = \omega r$, so the ratio of linear speeds is the ratio of angular speeds, or 640/423 = 1.51

$$v = \omega r = 67 \text{ rad/s} * 0.235 \text{ m} = 15.7 \text{ m/s}$$

$$a_{rad} = \omega^2 r = (67 \text{ rad/s})^2 * 0.235 \text{ m} = 1054 \text{ m} \big/ s^2 = 108 \text{ g}$$