

**PHYS 111**  
**OUTLINE OF SOLUTIONS**  
**CHAPTER 9 : Odd problems 31 – 71**

31. a) The moment of inertia of a bar about an axis perpendicular to the bar and passing through its center is :

$$I_{\text{bar}} = \frac{1}{12} M L^2$$

each ball contributes to the moment of inertia according to

$$m_{\text{ball}} r^2$$

where  $r$  is the distance from the rotation axis, so for this configuration :

$$I = \frac{1}{12} M L^2 + 2 m_{\text{ball}} r^2 = \frac{1}{12} 4 \text{ kg } (2 \text{ m})^2 + 2 (0.5 \text{ kg}) (1 \text{ m})^2 = 2.33 \text{ kg m}^2$$

b) If the axis is through one of the ends, we have :

$$I_{\text{bar}} = \frac{1}{3} M L^2$$

and one of the balls contributes nothing to the moment of inertia since its distance from the rotation axis is zero, then :

$$I = \frac{1}{3} M L^2 + m r^2 = \frac{1}{3} (4 \text{ kg}) (2 \text{ m})^2 + 0.5 \text{ kg } (2 \text{ m})^2 = 7.33 \text{ kg m}^2$$

c) The moment of inertia is zero since all the mass lies along the rotation axis.

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33. a) all the masses are equidistant from the rotation axis; if each side is 0.4 m, then the distance from the axis to each mass is :

$$r^2 = 0.2 \text{ m}^2 + 0.2 \text{ m}^2 = 0.08 \text{ m}^2$$

The total moment of inertia is then :

$$I = 4 (0.2 \text{ kg}) (0.08 \text{ m}^2) = 0.064 \text{ kg m}^2$$

b) All the masses are 0.2 m from the axis :

$$I = 4 * 0.2 \text{ kg } (0.2 \text{ m})^2 = 0.032 \text{ kg m}^2$$

c) two masses do not contribute to the moment of inertia since they are on the axis; the other two masses contribute (see part a to compute distance from axis) :

$$I = 2 (0.2 \text{ kg}) (0.08 \text{ m}^2) = 0.032 \text{ kg m}^2$$

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35.  $KE = \frac{1}{2} I \omega^2$

the moment of inertia of a uniform sphere is  $\frac{2}{5} M R^2$ , so

$$\text{KE} = \frac{1}{2} \cdot \frac{2}{5} M R^2 \omega^2$$

$$\omega = \frac{2\pi \text{ rad}}{27.3 \text{ days}} = \frac{2\pi \text{ rad}}{27.3 \text{ days} * 86400 \text{ s/day}} = 2.66 \times 10^{-6} \text{ rad/s}$$

$$\text{KE} = \frac{1}{2} \cdot \frac{2}{5} (7.35 \times 10^{22} \text{ kg}) (1.74 \times 10^6 \text{ m})^2 (2.66 \times 10^{-6} \text{ rad/s})^2 = 3.14 \times 10^{23} \text{ J}$$

37. 45 rpm = 4.71 rad/s

$$\text{KE} = \frac{1}{2} I \omega^2 \Rightarrow I = 2 \text{ KE} / \omega^2 = 2 * 0.25 \text{ J} / (4.71 \text{ rad/s})^2 = 0.0225 \text{ kg m}^2$$

The moment of inertia of a solid disc is :

$$I_{\text{disc}} = \frac{1}{2} M R^2 \Rightarrow M = 2 I / R^2$$

$$M = 2 * 0.0225 \text{ kg m}^2 / (0.3 \text{ m})^2 = 0.50 \text{ kg}$$

39.  $\Delta \text{KE} = \Delta \left( \frac{1}{2} I \omega^2 \right)$

If I is constant, then this becomes :

$$\Delta \text{KE} = \frac{1}{2} I \Delta (\omega^2) = \frac{1}{2} I (\omega_f^2 - \omega_o^2)$$

Solving for I :

$$I = \frac{2 \Delta \text{KE}}{(\omega_f^2 - \omega_o^2)} = \frac{-500 \text{ J} * 2}{(54.4 \text{ rad/s})^2 - (68 \text{ rad/s})^2} = 0.6 \text{ kg m}^2$$

41. The moment of inertia (see p. 279) of a hollow cylinder is :

$$I = \frac{1}{2} M (R_{\text{inner}}^2 + R_{\text{outer}}^2) = \frac{1}{2} M (R_1^2 + R_2^2)$$

We are given the values of the radii, but not of the mass of the cylinder. We have to compute this remembering that :

$$\rho = M / V = \frac{M}{\pi (R_2^2 - R_1^2) L} \Rightarrow M = \rho \pi L (R_2^2 - R_1^2)$$

where L is the length of the cylinder. Then, we can write :

$$I = \frac{1}{2} \rho \pi L (R_2^2 - R_1^2) (R_1^2 + R_2^2)$$

And the rotational kinetic energy is :

$$\text{KE} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left( \frac{1}{2} \rho \pi L (R_2^2 - R_1^2) (R_1^2 + R_2^2) \right) \omega^2$$

Solving for L :

$$L = \frac{4 \text{ KE}}{\rho \pi (R_2^2 - R_1^2) (R_1^2 + R_2^2) \omega^2} = \frac{4 \times 2.5 \times 10^6 \text{ J}}{2200 \frac{\text{kg}}{\text{m}^3} \cdot \pi \cdot (1.5^4 \text{ m}^4 - 0.5^4 \text{ m}^4) \cdot (2 \pi / 1.75 \text{ s})^2}$$

where I have rewritten

$$(R_2^2 - R_1^2) (R_1^2 + R_2^2) = R_2^4 - R_1^4$$

and the angular velocity is  $2 \pi \text{ rad} / 1.75 \text{ s}$

At this point you can plug in values and chug out the result.

43. While the problem does not state it exactly, we assume that the cylinder does not rise or fall; its potential energy does not change. We solve this problem using conservation of energy techniques applied to rotational motion.

At the start, there is no kinetic energy (the system starts from rest); the only energy is potential energy due to the height of the mass  $m$  ( $m = 1.5 \text{ kg}$ ) above its reference level. Thus, initially, we have :

$$U_i = m g h$$

At the end, the mass  $m$  has transferred all its initial PE to KE, but remember now that there is KE in both the motion of  $m$  and the rotation of  $M$  ( $M = 3.25 \text{ kg}$ ). At the end, we have :

$$\text{KE}_f = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

where  $v$  is the linear speed of  $m$  and  $\omega$  is the angular speed of  $M$ . The linear speed of the unwinding rope must be the same everywhere, so a point on the circumference of  $M$  must have linear speed  $v$ , which allows us to relate  $v$  and  $\omega$  :

$$v = \omega R \Rightarrow \omega = \frac{v}{R}$$

The moment of inertia of a disk is

$$I = \frac{1}{2} M R^2$$

putting all these together we have :

$$m g h = \frac{1}{2} m v^2 + \frac{1}{2} \left( \frac{1}{2} M R^2 \right) \omega^2 = \frac{1}{2} m v^2 + \frac{1}{4} M v^2$$

the last step following from the relationship  $\omega = v/R$

Solving for  $h$  :

$$h = \frac{\frac{1}{2} m v^2 + \frac{1}{4} M v^2}{m g} = \frac{0.5 * 1.5 \text{ kg} * (2.5 \text{ m/s})^2 + 0.25 * 3.25 \text{ kg} (2.5 \text{ m/s})^2}{1.5 \text{ kg} * 9.8 \text{ m/s}^2} = 0.66 \text{ m}$$

The angular velocity of the cylinder is

$$\omega = \frac{v}{R} = \frac{2.5 \text{ m/s}}{0.325 \text{ m}} = 7.69 \text{ rad/s}$$


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45. We have both translational and rotational KE :

$$\text{KE} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

where  $v$  is the speed of the center of mass and  $\omega$  is the angular velocity. In this case, we have :

$$I = \frac{1}{2} M R^2$$

since  $v = R \omega$  :

$$\text{KE} = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{1}{2} m R^2 \omega^2 = \frac{1}{2} m v^2 + \frac{1}{4} m v^2 = \frac{3}{4} m v^2$$

For the values here :  $r = 0.5 \text{ m}$ ,  $m = 75 \text{ kg}$ ;  $\omega = 0.5 \text{ rev/s} = \pi \text{ rad/s}$  so  $v = \pi/2 \text{ m/s}$  :

$$\text{KE} = \frac{3}{4} \cdot 75 \text{ kg} \cdot (\pi/2 \text{ rad/s})^2 = 139 \text{ J}$$

The fraction of rotational KE to total KE is :

$$\frac{\text{rotational KE}}{\text{total KE}} = \frac{\frac{1}{4} m v^2}{\frac{3}{4} m v^2} = \frac{1}{3}$$


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47. The phrase 'rolling without slipping' means the relationship  $v = R \omega$  is valid (if there is slipping, then there is motion of the center of mass without rotation).

$$v = R \omega = 0.6 \text{ m} \cdot 3 \text{ rad/s} = 1.8 \text{ m/s}$$

for a hoop,  $I = M R^2$  :

$$\text{KE} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} m R^2 \omega^2 = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 = m v^2$$

$$\text{KE} = 2.2 \text{ kg} \cdot (1.8 \text{ m/s})^2 = 7.13 \text{ J}$$


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49. We combine conservation of energy with concepts of rotational motion. At the base of the hill, there is no potential energy, and both both translational and rotational kinetic energy. At the top of the slope, there is potential energy but no kinetic. Equating energies (note there are no losses to dissipative forces) :

$$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = m g h$$

in this problem :  $m = 0.426 \text{ kg}$ ;  $r = 0.113 \text{ m}$ ;

$$I = \frac{2}{3} m R^2 \text{ (see p. 279 for moment of inertia of thin hollow sphere)}$$

so we have :

$$\frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{2}{3} m R^2 \omega^2 = m g h$$

$$\frac{1}{2} m v^2 + \frac{1}{3} m v^2 = m g h$$

$$\frac{5}{6} m v^2 = m g h \Rightarrow v = \sqrt{6 g h / 5} = \sqrt{\frac{6}{5} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 5 \text{ m}} = 7.67 \text{ m/s}$$

$$v = R \omega \Rightarrow \omega = \frac{v}{R} = \frac{7.67 \text{ m/s}}{0.113 \text{ m}} = 67.9 \text{ rad/s}$$

$$\text{KE}_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{2}{3} m R^2 \omega^2 = \frac{1}{3} \cdot 0.426 \text{ kg} \cdot (0.113 \text{ m} \cdot 67.9 \text{ rad/s})^2 = 8.35 \text{ J}$$

51. In each case, we have :

$$\text{KE}_{\text{tot}} = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

and we can write moments of inertia in a general way :

$$I = f M R^2$$

where  $f$  is a coefficient that varies with the geometry of the object (for a hoop  $f = 1$ , for a sphere  $f = 2/5$ , etc.), and if there is no slipping :

$$v = R \omega$$

thus :

$$\text{KE}_{\text{total}} = \frac{1}{2} M v^2 + \frac{f}{2} M R^2 \omega^2 = \frac{1}{2} (1 + f) M v^2$$

$$\text{KE}_{\text{rot}} = \frac{f}{2} M v^2$$

$$\frac{\text{KE}_{\text{rot}}}{\text{KE}_{\text{tot}}} = \frac{\frac{f}{2} M v^2}{\frac{1}{2} (1 + f) M v^2} = \frac{f}{1 + f}$$

so, for a uniform solid cylinder,  $f = 1/2$  and :

$$\frac{\text{KE}_{\text{rot}}}{\text{KE}_{\text{tot}}} = \frac{1/2}{3/2} = \frac{1}{3}$$

for a uniform sphere,  $f = 2/5$  :

$$\frac{\text{KE}_{\text{rot}}}{\text{KE}_{\text{tot}}} = \frac{2/5}{7/5} = \frac{2}{7}$$

for a thin walled hollow sphere,  $f = 2/3$  :

$$\frac{\text{KE}_{\text{rot}}}{\text{KE}_{\text{tot}}} = \frac{2/3}{5/3} = \frac{2}{5}$$

for a hollow cylinder with inner and outer radii of  $R/2$  and  $R$  :

$$I = \frac{1}{2} M (R_1^2 + R_2^2) = \frac{1}{2} M \left( \left( \frac{R}{2} \right)^2 + R^2 \right) = \frac{5}{8} M R^2$$

so here,  $f = 5/8$  and :

$$\frac{KE_{\text{rot}}}{KE_{\text{tot}}} = \frac{5/8}{13/8} = \frac{5}{13}$$

53. This is a conservation of energy problem with several parts. At the top of the incline, there is only the potential energy of the wheels/boat/cart assembly. At the bottom, there is no potential energy; the kinetic energy is in the form of the rotational motion of the wheels, and also the translational motion of the wheels and also of the cart and the boat. Remember that only the wheels rotate. However, the wheels also have translational kinetic energy. The boat and cart do not rotate, so have translational kinetic energy and are moving at the same linear speed as the center of mass of the wheels. So, at the top of the incline :

$$U_i = (4 m_w + m_c + m_b) g h$$

where :

$m_w$  = mass of each wheel;  $m_c$  = mass of cart;  $m_b$  = mass of boat

$h$  = height above ground =  $L \sin \theta = 16 \sin 7.5 = 2.09$  m

Each wheel is a solid sphere, so the total moment of inertia of the wheels is :

$$I = \frac{2}{5} (4 m_w) R^2 = \frac{8}{5} m_w R^2$$

where  $v$  is the linear speed of the center of mass of the wheels. The total kinetic energy at the bottom of the ramp is :

$$\begin{aligned} KE_{\text{tot}} &= \frac{1}{2} I \omega^2 + \frac{1}{2} (4 m_w + m_b + m_c) v^2 = \frac{1}{2} \left( \frac{8}{5} m_w R^2 \right) \omega^2 + \frac{1}{2} (4 m_w + m_b + m_c) v^2 = \\ &= \frac{4}{5} m_w v^2 + \frac{1}{2} (4 m_w + m_b + m_c) v^2 \end{aligned}$$

Equating energies :

$$(4 m_w + m_c + m_b) g h = \frac{4}{5} m_w v^2 + \frac{1}{2} (4 m_w + m_b + m_c) v^2$$

Substituting values :

$$\begin{aligned} (4 \cdot 45 \text{ kg} + 150 \text{ kg} + 750 \text{ kg}) (9.8 \text{ m/s}^2) (2.09 \text{ m}) &= \\ (0.8 \cdot 45 \text{ kg} + 0.5 \cdot (180 \text{ kg} + 150 \text{ kg} + 750 \text{ kg})) v^2 &= \\ 22\,120 \text{ J} = 576 v^2 \Rightarrow v &= 6.20 \text{ m/s} \end{aligned}$$

55. The language of the problem suggests that the maximum size of the cylindrical shaft is 16 cm across, or a maximum radius of 0.08 m. Then, to find the angular acceleration of the shaft, we have :

$$a = R \alpha$$

where  $a$  is linear acceleration;  $R$  is the radius of the cylinder, and  $\alpha$  is the angular acceleration. For the values here :

$$\alpha = \frac{a}{R} = \frac{0.15 * 9.8 \text{ m/s}^2}{0.08 \text{ m}} = 18.4 \text{ rad/s}^2$$

The weight of the elevator is irrelevant.

57. Oh good, let's use English units. A point on the edge of the tire travels a total distance of  $2\pi R$  for each revolution of the tire. So for a tire diameter of 24 inches (2 feet), the radius is 1 foot and the linear distance traveled by a car during one tire revolution is  $2\pi$  feet. To travel 0.1 mi ( 528) feet, the tires need to make :

$$\frac{528 \text{ ft}}{2\pi \text{ ft/rev}} = 84 \text{ revs}$$

$$5000 \text{ revs} * 2\pi \text{ ft/rev} * 1 \text{ mile} / 5280 \text{ ft} = 5.95 \text{ miles}$$

c) In this case, you will have driven 28/24 of the recorded distance, or 583 mi.

59. a) According to the statement of the problem, the scale speed is

$$v_{\text{scale}} = v_{\text{toy}} (\text{length of car} / \text{length of toy})$$

$$\text{or } v_{\text{toy}} = 700 \text{ km/h} * 0.15 \text{ m} / 3 \text{ m} = 35 \text{ km/h}$$

Converting units :

$$35 \text{ km/h} = 35 \text{ km/h} * 1000 \text{ m/km} * 1 \text{ h} / 3600 \text{ s} = 9.72 \text{ m/s}$$

b) the KE of a mass of 0.18 kg moving with this speed is :

$$\text{KE} = \frac{1}{2} m v^2 = 0.5 * 0.18 \text{ kg} * (9.72 \text{ m/s})^2 = 8.51 \text{ J}$$

c) The angular velocity needed to store this amount of KE is :

$$\text{KE} = \frac{1}{2} I \omega^2 \Rightarrow \omega = \sqrt{2 \frac{\text{KE}}{I}} = \sqrt{\frac{2 * 8.51 \text{ J}}{4 \times 10^{-5} \text{ kg m}^2}} = 652 \text{ rad/s}$$

(Remember that the problem tells you the moment of inertia of the flywheel storing the energy).

61. First, we know that rotational kinetic energy is :

$$\frac{1}{2} I \omega^2$$

but we have to give some thought about our choice of  $I$ . In the first case, we think of the Earth as a point particle of mass  $M$  (the mass of the Earth) moving along a circular (close enough to circular path for this problem) arc of radius  $R$  (where  $R$  is the earth - sun distance). For this problem, we have :

$$I = M R^2 \text{ and } \omega = 1 \text{ rev/year} = 2\pi \text{ rad} / 3.15 \cdot 10^7 \text{ s}$$

(a mnemonic used by astronomers : the number of seconds in a year is  $\pi 10^7$  s, so that the angular velocity of the Earth is  $2 \cdot 10^{-7}$  rad/s). With these values:

$$\text{KE due to annual motion} = \frac{1}{2} (6.0 \times 10^{24} \text{ kg}) (1.5 \times 10^{11} \text{ m})^2 (2 \times 10^{-7} \text{ rad/s})^2 = 2.7 \times 10^{33} \text{ J}$$

When considering the Earth's daily motion, we regard the Earth as a uniform sphere of radius  $r$  and angular velocity  $\omega = 2 \pi$  rad/day, so :

$$\text{KE}_{\text{rotating Earth}} = \frac{1}{2} \cdot \frac{2}{5} M r^2 \omega^2 = \frac{1}{5} (6 \times 10^{24} \text{ kg}) (6.4 \times 10^6 \text{ m})^2 (2 \pi \text{ rad} / 86400 \text{ s})^2 = 2.60 \times 10^{29} \text{ J}$$

There is approximately 10,000 times more angular momentum in the Earth's annual orbit than in our daily rotation. All hail lever arm!

63. If the belt is not slipping, all points on the surface of the belt have the same linear speed. We can compute the linear speed of points on the motor shaft from  $v = r \omega$

Here,  $\omega = 60 \text{ rev/s} = 60 \cdot 6.28 \text{ rad/s} = 376.8 \text{ rad/s}$ . The radius of the motor shaft = 0.0045 m, so  $v = 0.0045 \text{ m} \cdot 376.8 \text{ rad/s} = 1.70 \text{ m/s}$ .

The belt has the same linear velocity at the wheel (whose radius is 2 cm), so we find  $\omega$  from :

$$\omega = \frac{v}{R} = \frac{1.70 \text{ m/s}}{0.02 \text{ m}} = 84.8 \text{ rad/s}$$

65. The point here is that the rotational axis runs through your head, body, and legs and your arms extend out. We estimate the total moment of inertia by approximating the individual moments of inertia of your head, torso, legs/trunk and arms. So, consider that your head is a sphere of radius  $R_h$  (in no other context could a professor say this to students and not get in trouble), your trunk is a cylinder of radius  $R_t$ , and your arms are rods of length  $L$ . The total moment of inertia is :

$$I_{\text{total}} = I_{\text{head}} + I_{\text{torso/legs}} + I_{\text{arms}}$$

$$I_{\text{head}} = \frac{2}{5} (0.07 M) (R_h^2)$$

$$I_{\text{torso}} = \frac{1}{2} (0.8 M) (R_t^2)$$

$$I_{\text{arms}} = \frac{1}{3} (0.065 M) L^2 * 2$$

$M$  is the total mass of the body, the coefficients of  $M$  are taken from the data given in the problem; each arm is 0.065 times your body mass and I compute the moment for both arms. So, for "typical?" values :

Let's say a 5' 6" (1.68m) tall figure skater has a mass of 60 kg. Assume the head is 20 cm in diameter (that's what I measure the length of my head...the width is less but I am not going to start using oblate spheroidal coordinates at 4:25 am to solve this stupid problem. ) Let's also assume this is half the radius of their torso, and that the length of their arms with respect to the rotation axis is 0.7m. We have for a total moment of inertia:



$$I = 0.4 * 0.07 * 60 \text{ kg } (0.1 \text{ m})^2 + 0.5 * 0.8 * 60 \text{ kg } (0.2 \text{ m})^2 + 0.33 * 0.13 * 60 \text{ kg } (0.7 \text{ m})^2$$

$$I = 2.24 \text{ kg m}^2$$

Your book reports an answer 50 % greater than this but that is easily explainable by differences in assumptions about body parameters (they do not state theirs). Also, you could argue that you should consider your head as a cylinder, but the difference is small, (f as defined in my solution to problem 49 above is 0.4 for a sphere and 0.5 for a cylinder, and this is a small difference compared to other estimates we make for this problem). The point here is not to obtain a precise number, but to show how you can estimate the moment of inertia. Now, in the next chapter, we will ask what happens when the skater pulls her/his arms in (this lowers the moment of inertia, and because of conservation of angular momentum, will spin faster).

67. The key here is that the rotation axis is perpendicular to the plane in which the rod lies. Therefore, each half of the "V" can be considered a thin rod of length  $L/2$  and mass  $M/2$  rotating about its end. We have :

$$I = 2 * \frac{1}{3} \left( \frac{M}{2} \right) \left( \frac{L}{2} \right)^2 = \frac{1}{12} M L^2$$

remember that the moment of inertia of a rod around its end is  $1/3 M L^2$ ; the factor of two takes into account the two branches of the "V". Notice that the result is the same as we would get for a rod of length  $L$  and mass  $M$  rotating about its center. Substituting the numbers from the book :  $M = 0.32 \text{ kg}$  and  $L = 0.5 \text{ m}$  yields the answer in the back of the book,  $0.00667 \text{ kg m}^2$

69. What I think the problem is saying is that the stone falls a vertical distance of  $H$  and then gets launched by the upward curving hill. a) If there is no friction, the stone will not roll and will only slide. Therefore it has only translational KE at the bottom; thus, all its original PE has gone into translational KE at the bottom. If there is no friction, all this will be converted to PE, and the stone will rise to its original height of  $H$ . b) Here, there is friction so the stone rolls and a part of its PE goes into rotational KE and part into translational KE. It is only the translational KE that will be converted to gravitational PE upon launch. In problem 49, we showed that  $2/7$  of the total kinetic energy of a rolling sphere is rotational KE, or that  $5/7$  is in translational. Therefore, the ball will rise to  $5/7 H$ , since some of the initial PE is in the form of rotational energy.

71. a) The arms travel  $1/6$  of a circle (60 degrees) in a second, so the angular velocity is  $2 \pi/6 = \pi/3 \text{ rad/s}$  (1.05 rad/s).

b) Use the data in the problem in association with :

$$KE = \frac{1}{2} I \omega^2$$

In both cases, we are modelling the body parts as rods, where :

$$I_{\text{rod}} = \frac{1}{3} M L^2$$

so, for the arms :

$$I_{\text{arm}} = \frac{1}{3} (0.13 * 75 \text{ kg}) (0.7 \text{ m})^2 = 1.59 \text{ kg m}^2$$

$$I_{\text{legs}} = \frac{1}{3} (0.37 * 75 \text{ kg}) (0.9 \text{ m})^2 = 7.49 \text{ kg m}^2$$

so :

$$KE_{\text{arms}} = \frac{1}{2} * 1.59 \text{ kg m}^2 * (1.05 \text{ rad/s})^2 = 0.87 \text{ J}$$

$$KE_{\text{legs}} = \frac{1}{2} * 7.49 \text{ kg m}^2 * (1.05 \text{ rad/s})^2 = 4.13 \text{ J}$$

or a total rotational kinetic energy of 5 J.

$$\text{c) translational KE} = \frac{1}{2} M V^2 = \frac{1}{2} * 75 \text{ kg} * (1.39 \text{ m/s})^2 = 72.3 \text{ J}$$

where I have taken the liberty of converting the velocity of 5 km/h to 1.39 m/s.

73. This is a great problem, but somehow your book reports a numerical answer when it uses only symbolic variables. But here's how it works. This problem involves concepts of energy conservation, work done by dissipative forces, and kinetic energy of rotation. We will use an expanded form of text equation 7.16, to include energy of rotation :

$$U_f + KE_f = U_i + KE_i + W_{\text{other}}$$

In essence, the total energy we have at the start will go into EITHER energy of the masses at the end, OR into work done against friction. Now let's apply this simple idea to this situation.

At the start, the only energy is the grav potential energy of mass B, so that :

$$U_i = m_B g d$$

$$KE_i = 0$$

At the end, there is translational KE of masses A and B; the pulley of moment of inertia I is rotating, and work was done against friction. Since the rope doesn't slip, the linear velocity is the same everywhere, so the final speed of A is equal to the final speed of B, and this is also the linear speed of a point on the circumference of the wheel.

If the mass B falls a distance d, the mass A moves a distance d along the table, so we have after B has fallen a distance d :

$$KE \text{ of mass B} = \frac{1}{2} m_B v^2$$

$$KE \text{ of mass A} = \frac{1}{2} m_A v^2$$

$$\text{KE of pulley} = \frac{1}{2} I \omega^2 = \frac{1}{2} I \left( \frac{v}{R} \right)^2 = \frac{1}{2} \frac{I}{R^2} v^2$$

we are not told the shape of the pulley, so we can't simplify this expression any more.

Since mass A moves along a surface with friction, the work done against friction is :

$$W_{\text{other}} = -\mu m_A g d$$

Combining all this, we have :

$$m_B g d - \mu m_A g d = \frac{1}{2} m_B v^2 + \frac{1}{2} m_A v^2 + \frac{1}{2} \frac{I}{R^2} v^2$$

Combining terms :

$$v = \sqrt{\frac{2(m_B - \mu m_A) g d}{m_B + m_A + I/R^2}}$$