PHYS 111 HOMEWORK #11

Due date: You have a choice here. You can submit this assignment on Tuesday, 2 December and receive a 20 % bonus, or you can submit this for normal credit on Thursday, 4 December. If I receive your homework on Tuesday, I will return it Thursday. If I receive it Thursday, you can retrieve it in office hours on either Friday (5 December) or Monday (8 December). Remember the exam is on Tuesday the 9 th.

All are from the "problems" sections of the text.

1. Problem 10, p. 287

Solution : The two hands move at different rates that we will call $\omega_1 = 0.0425$ rad/s and $\omega_2 = 0.0163$ rad/s. We are asked to determine how long it will take for the faster hand to catch up with or "lap" the slower hand. Since the faster hand moves 2.6 times faster than the slower, we know that the faster hand will complete one full revolution while the slower hand is still completing its first revolution.

We want to find the time when the two hands make the same angle with respect to the vertical. In other words, we want :

$$\theta_1(t) = \theta_2(t)$$

The slower hand will move through an angle given by :

$$\theta_2(t) = \omega_2 t$$

We dont't want to know the total angle traveled by the faster hand; we want to know the angle it makes with respect to the vertical. Therefore, we want to compute how far (in radians) it has traveled after having completed one full revolution. To do this, we compute:

$$\theta_1(t) = \omega_1(t-t_o)$$

where t_o is the time to complete one revolution, so that $(t - t_o)$ is the time interval after having completed one revolution. The time to complete one full revolution is:

$$t_{\rm o} = \frac{2\pi}{\omega_1}$$

so that we have :

$$\theta_1(t) = \omega_1 \left(t - \frac{2\pi}{\omega_1} \right) = \omega_1 t - 2\pi$$

the two hands are at the same angle when :

$$\omega_1 t - 2\pi = \theta_2 (t) = \omega_2 t \Rightarrow t = \frac{2\pi}{\omega_1 - \omega_2} = \frac{2\pi}{0.0425 \text{ rad/s} - 0.0163 \text{ rad/s}} = 240 \text{ s}$$

It takes 240 s for the faster hand to catch up with the smaller hand, so the next occurence will be at t = 480 s.

2. Problem 18, p. 288 *Solution :* a) We use the basic equations :

average angular velocity =
$$\omega_{av} = \frac{\omega_f + \omega_o}{2}$$

and

angular displacement = $\theta = \omega_{av} t$

Combining these :

$$\theta = \frac{1}{2} (\omega_{o} + \omega_{f}) t \Rightarrow \omega_{o} = \frac{2\theta}{t} - \omega_{f} = \frac{2 \cdot 162 \text{ rad}}{4 \text{ s}} - 108 \text{ rad/s} = -27 \text{ rad/s}$$

b) $\alpha = \frac{\omega_{f} - \omega_{o}}{t} = \frac{108 \text{ rad/s} - (-27 \text{ rad/s})}{4 \text{ s}} = 33.8 \text{ rad/s}^{2}$

3. Problem 26, p. 288

Solution : a) At the beginning the angular and linear velocities are zero, so the centripetal accel is zero. The tangential acceleration is :

$$a_{tan} = r \alpha = 0.3 \text{ m} \cdot 0.6 \text{ rad} / \text{s}^2 = 0.18 \text{ m} / \text{s}^2$$

and this is the resultant acceleration.

b) The tangential acceleration is constant. To find the radial acceleration, we have to find the angular velocity, which we compute from :

$$\omega_{\rm f}^2 = \omega_{\rm o}^2 + 2\,\alpha\,\theta$$

Since the wheel starts from rest, this gives us :

$$\omega_{\rm f}^2 = 2 \alpha \theta = 2 \cdot 0.6 \, \text{rad} / {\rm s}^2 \cdot \pi / 3 \, \text{rad} = 1.26 \, \text{rad}^2 / {\rm s}^2$$

and the centrip accel is :

$$a_{rad} = \omega^2 r = 0.38 \,\mathrm{m/s^2}$$

The total acceleration is given by the Pythagorean theorem :

$$a_{total} = \sqrt{a_{tan}^2 + a_{rad}^2} = \sqrt{(0.18 \text{ m}/\text{s}^2)^2 + (0.38 \text{ m}/\text{s}^2)^2} = 0.42 \text{ m}/\text{s}^2$$

The direction of the acceleration vector with respect to the radial line is given by :

$$\tan \theta = \frac{a_{tan}}{a_{rad}} = \frac{0.18 \text{ m/s}^2}{0.38 \text{ m/s}^2} \Rightarrow \theta = 25.3 \text{ degrees.}$$

c) For this part, the tangential acceleration is the same, and the radial acceleration is doubled since the angular displacement is doubled. This yields a total acceleration of 0.78 m/s^2 making an angle

with the radial line of:

$$\tan \theta = \frac{0.18 \text{ m/s}^2}{0.76 \text{ m/s}^2} \Rightarrow \theta = 13.3 \text{ degrees}$$

4. Problem 42, pp. 289 - 290

Solution : We begin by treating this as an energy conservation problem. Initially, there is no kinetic energy and the stone (of mass m) has potential energy due to its position 2.5 m above the reference level for this problem. At the end, there is no potential energy (the stone having reached the local "zero height" for this problem), but the stone has translational kinetic energy and the cylinder (of radius R and mass M) has rotational kinetic energy. Since there are no dissipative forces in this case, our energy conservation equation yields :

$$U_{i} + K_{i} = U_{f} + K_{f}$$
$$m g h = \frac{1}{2} m v^{2} + \frac{1}{2} I \omega^{2}$$

Making use of the relations :

$$I_{cylinder} = \frac{1}{2} M R^2$$
 and $v = \omega R \Rightarrow \omega = \frac{v}{R}$

we get :

$$mgh = \frac{1}{2}mv^{2} + \frac{1}{2}\left(\frac{1}{2}MR^{2}\right)\left(\frac{v}{R}\right)^{2} = \frac{1}{2}\left(m + \frac{M}{2}\right)v^{2}$$

Solving for M :

M = 2 m
$$\left[\frac{2 \text{ g h}}{v^2} - 1\right] = 2 \cdot 3 \text{ kg} \left[\frac{2 \cdot 9.8 \text{ m}/\text{s}^2 \cdot 2.5 \text{ m}}{(3.5 \text{ m}/\text{s})^2} - 1\right] = 18 \text{ kg}$$

5. Problem 52, p. 290

Solution : This problem begins as the previous one, as an energy conservation problem. In this case, the moment of inertia of a hoop is :

$$I_{hoop} = M R^2$$

so our energy conservation equation is :

Mgh =
$$\frac{1}{2}$$
 Mv² + $\frac{1}{2}$ I ω^2 = $\frac{1}{2}$ Mv² + $\frac{1}{2}$ MR² $\left(\frac{v}{R}\right)^2$ = Mv²

and the linear speed of the hoop is :

$$v = \sqrt{gh} = \sqrt{9.8 \frac{m}{s^2} \cdot 0.75 m} = 2.71 m/s$$

The angular velocity is :

$$\omega = \frac{v}{R} = \frac{2.71 \text{ m/s}}{0.08 \text{ m}} = 33.9 \text{ rad/s}$$

6. Problem 70, p. 292

Solution : The first part of this problem is a conservation of energy problem in which we find the translational speed of the ball at the top of the hill. This will be the horizonatal speed of the ball that we will use to determine its time of flight and range from the base of the hill. Using conservation of energy with rotation, we have :

$$U_i + K_i = U_f + K_f$$

At the bottom of the hill, there is translational and rotational kinetic energy, but no potential. At the top, there will be gravitational potential energy, and still some translational and rotational kinetic energy. Using the values given (and the moment of inertia of a sphere), we obtain :

$$\frac{1}{2} m v_o^2 + \frac{1}{2} I \omega_o^2 = m g h + \frac{1}{2} m v_T^2 + \frac{1}{2} I \omega_T^2$$

where subscripts "o" refer to initial conditions and "T" refer to values at the top of the hill. Using the moment of inertia of a solid sphere :

$$I_{\text{sphere}} = \frac{2}{5} \text{ M R}^2,$$

and the relationship between v and ω for an object that rolls without slipping :

$$v = \omega R$$

we get :

$$\frac{1}{2} m v_o^2 + \frac{1}{2} \left(\frac{2}{5} m R^2\right) \left(\frac{v_o}{R}\right)^2 = m g h + \frac{1}{2} m v_T^2 + \frac{1}{2} \left(\frac{2}{5} m R^2\right) \left(\frac{v_o}{R}\right)^2$$

Collecting terms and cancelling the common factor of m :

$$\frac{7}{10} v_o^2 = g h + \frac{7}{10} v_T^2$$

or :

$$v_T^2 = v_o^2 - \frac{10}{7} g h \Rightarrow v_T^2 = (25 m/s)^2 - \frac{10}{7} \cdot 9.8 m/s^2 \cdot 28 m = 233 (m/s)^2$$

or $v_T = 15.3 \text{ m/s}$

This will be the horizontal speed of the ball as it travels off the cliff. The second part of the problem is familiar to us from Chapter 3. We know that in the absence of air friction, the horizontal velocity will remain constant, and the vertical velocity of the ball will increase in magnitude due to the force of gravity. The total distance traveled will be the time of flight multiplied by the horizontal speed :

range = time of flight · horizontal speed =
$$\sqrt{\frac{2 h}{g}} \cdot v_T = 36.5 m$$

The final speed of the object will be :

$$v_{\text{final}} = \sqrt{v_x^2 + v_y^2}$$

We know the horizontal velocity, and the vertical velocity will be simply g t, where t is the time of flight determined above :

$$v_{vertical} = gt = g\sqrt{\frac{2h}{g}} = \sqrt{2hg} = 23.4 \text{ m/s}$$

 $v_{final} = \sqrt{233 (\text{m/s})^2 + 549 (\text{m/s})^2} = 28 \text{ m/s}$

Of course total energy is conserved. There is no requirement that total translational kinetic energy is conserved; rather the total of all forms of energy is conserved. Initially, some of the energy was in the form of rotational kinetic. At the end, all the energy was translational; the total energy is conserved, but the initial rotational has been converted to translationa.

7. Problem 4, p. 323

Solution : For each force, we have to find the product of the force times moment arm. To find the total torque, we need to determine the direction and therefore sign of each torque, and sum the vector quantities.

Let's start with two easy cases : The action line of force B goes through the point P; therefore B has no moment arm with respect to P and therefore generates no torque around P. Force D is at right angles to the 20 cm line, so it is easy to see that the moment arm of force D is 20 cm, and therefore the torque due to D around P is :

$$|\tau_{\mathbf{D}}| = |\mathbf{F}| \mathbf{L} = 50 \,\mathrm{N} \cdot 0.2 \,\mathrm{m} = 10 \,\mathrm{N} \cdot \mathrm{m}$$

This force produces a clockwise torque around P, and I will call this the positive direction.

Now let's look at force C. The component of force C that acts in the direction of D is C cos 60, so we can conclude that C exerts a clockwise torque around P of C cos 60. We can also use the formula :

$$\tau = Fr \sin \theta$$

where θ is the angle between the force vector and the position vector between P and the force. The angle between the 20 cm line and force C is 150 degrees, so we could write :

$$\tau_{\rm C} = 50 \,{\rm N} \cdot 0.2 \,{\rm m} \sin 150 = 5.00 \,{\rm n} \cdot {\rm m}$$

and this is the same result as F L cos 60 (and is also positive)

The component of force A perpendicular to the radius line is A cos 30, so we can express its torque as :

$$\tau_{\rm A} = 50 \,{\rm N} \cdot 0.2 \,{\rm m} \cos 30 = -8.66 \,{\rm N} \cdot {\rm m}$$

or as :

$$\tau_{\rm A} = \operatorname{Fr} \sin \theta = 50 \,\mathrm{N} \cdot 0.2 \,\mathrm{m} \sin 120 = -8.66 \,\mathrm{N} \cdot \mathrm{m}$$

I include the minus sign since this force will produce a counterclockwise torque. The total torque is

the sum of these individual torques, and is :

$$\tau_{\text{total}} = 10 \text{ N} \cdot \text{m} + 5 \text{ N} \cdot \text{m} - 8.66 \text{ N} \cdot \text{m} = 6.34 \text{ N} \cdot \text{m}$$

8. Problem 12, p. 324

Solutions : If we know the string is unwinding at a speed of 3.5 m/s when the suitcase hits the floor, we also know that a point on the rim of the wheel has a tangential velocity of 3.5 m/s. Its angular velocity is given by :

$$\omega = \frac{v}{R} = \frac{3.5 \text{ m/s}}{0.4 \text{ m}} = 8.75 \text{ rad/s}$$

We can find the acceleration of the wheel by writing Newton's laws for the system :

for the suitcase, Newton's second law is :

$$\Sigma \mathbf{F} = \mathbf{T} - \mathbf{m} \mathbf{g} = -\mathbf{m} \mathbf{a}$$

(if we define the direction of the tension as positive, then both the direction of gravity and the acceleration of the mass are negative). The sum of torques gives us :

$$\Sigma \tau = TR = I\alpha$$

here, the only torque acting on the wheel is the tension, which acts at a distance R from the rotational axis. Solving for T in the force equation gives us :

$$T = m(g - a)$$

and we can use the information provided to find the acceleration :

$$v_{f}^{2} = v_{o}^{2} + 2 a s \Rightarrow a = \frac{v_{f}^{2}}{2 s} = \frac{(3.5 \text{ m/s})^{2}}{2 \cdot 4 \text{ m}} = 1.53 \text{ m/s}^{2}$$

Now, we know that angular and linear accelerations are related :

$$\alpha = \frac{a}{R}$$

so the torque equation becomes :

$$TR = I\alpha \Rightarrow I =$$

$$\frac{\mathrm{TR}}{\alpha} = \frac{\mathrm{TR}}{\mathrm{a/R}} = \frac{\mathrm{TR}^2}{\mathrm{a}} = \frac{\mathrm{m}(\mathrm{g}-\mathrm{a})\mathrm{R}^2}{\mathrm{a}} = \frac{15\mathrm{kg}(9.8\mathrm{m/s}^2 - 1.53\mathrm{m/s}^2)(0.4\mathrm{m})^2}{1.53\mathrm{m/s}^2} = 13\mathrm{kgm}^2$$

9. Problem 15, p. 324.

Solution : The basic equation here is the torque equation :

$$\Sigma \tau = I \alpha$$

The problem requires us to compute the moment of inertia of the object around its rotational axis and also the angular acceleration. The moment of inertia of this object can be written as :

$$I_{total} = I_{shell} + I_{masses}$$

The moment of inertia of a spherical shell is known, and is

$$I_{shell} = \frac{2}{3} M R^2$$

and each mass will contribute an amount equal to $m r^2$ where m is the mass and r the distance from the rotation axis. Two of the balls lie on the rotation axis so contribute nothing to the moment of inertia, and the other two balls add m R^2 each, so the total moment of inertia of the system is :

$$I_{\text{total}} = \frac{2}{3} \text{ M R}^2 + 2 \text{ m R}^2 = 0.667 (8.4 \text{ kg}) (0.25 \text{ m})^2 + 2 (2 \text{ kg}) (0.25 \text{ m})^2 = 0.60 \text{ kgm}^2$$

The angular acceleration is found from :

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega_{\rm f} - \omega_{\rm o})}{t} = \frac{5.23 \, {\rm rad/s} - 7.85 \, {\rm rad/s}}{30 \, {\rm s}} = -0.087 \, {\rm rad/s^2}$$

where I have converted the angular velocities from rpm to rad/s. Finally, the torque needed to slow this system is :

$$\tau = I \alpha = -0.052 \,\mathrm{N} \cdot \mathrm{m}$$