PHYS 111
HOMEWORK #2--Solutions

1. The sorts of estimates and order of magnitude problems covered in your text are often known as Fermi problems, named after the Univ. of Chicago physics professor Enrico Fermi who had a genius (for among many other things) estimating the answers to seemingly complicated questions. Your Fermi problem is : Estimate the number of vampires in Chicago.

Your solution must explicitly describe the assumptions you are making about the behavior of vampires, the data you are using, and the logic that leads to your result. Since there is no well defined answer to this question, your grade will depend solely on your logic and explanation. (If you have written more than two pages of explanation, that is likely too much.)

Solution : We start by stating our assumptions. It is a bit difficult to make reasoned assumptions when there are almost no data, but our assumptions should be consistent with the one observation we have, namely, there is no evidence for the existence of vampires. From this we conclude that either vampires don' t exist at all, or in such small numbers that they are below our detection capabilities. There are many ways one could proceed, but I will assume that a) vampires remain hidden from humans; b) vampires must interact with humans; c) vampires do so by feeding on humans with a certain frequency; d) humans die each time there is a human - vampire interaction; e) I further assume that vampires do not die nor are killed during human - vampire interactions.

Now we need to determine the frequency at which vampires feed, and how many human deaths can be attributed to vampire-human interactions. I will assume that each vampire feeds once/day (actually, night). If vampires feed less frequently, we simply scale our final result accordingly (e.g., if vampires feed weekly, then scale the number we obtain here by multiplying by seven). One place to start might be the number of murders in Chicago. In 2013, there were (http://www.dnainfo.com/chicago/2013-chicago-murders/timeline?mon=1) 421 murders. The link in parentheses lists each murder with some details. As you review this site, you will realize that most of these murders are not likely attributable to vampires since there were witnesses, the murders occurred during daytime hours, and the majority of these murders were caused by shooting. During a random review of roughly 1/4 the total murders during 2013, the reference shows that 104 of 120 murders were the result of shooting. If all the other murders (13%) are the result of vampires, then there were 56 vampire related murders in Chicago in 2013.

Are there other possible data sets to consult? There are many more missing persons than murders. According to FBI statistics (http://www.fbi.gov/about-us/cjis/ncic/ncic-missing-person-and-unidentified-person-statistics-for-2013), 627 thousand individuals went missing in the United States last year. If we assume Chicago accounted for 1% of these (roughly the proportion of Chicago's population in the United States) then 6270 individuals went missing last year in Chicago. If all of these were the result of vampire activity, adding the number of vampire related murders yields a total of 6326 vampire related deaths in 2013. If vampires feed nightly, then we can conclude as an upper estimate that there are no more than 6326 missing persons/365 days/yr / 1 person/vampire/day = 17 vampires in Chicago in 2013.

Another assumption (made in a paper published in 2006) is that each vampire-human interaction turns the human into a vampire. To show this is unlikely, consider very simple cases. If we start with but one vampire who feeds once per month (the assumption made in the paper), then at the end of the first month there are two vampires. Each month, the number of vampires doubles (assuming each vampire finds its own prey). Thus, the number of vampires in the population can be described by 2^n where n is the number of months after the first vampire incident. If n = 10, there are approximately 1000 vampires; for n = 20 approximately 10^6, and for n = 40 (just over 3 years) we would obtain the impossible result of 10^{12}vampires, much larger than the total population of human on the planet.

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2. Estimate the mass of gas that you inhale during a night of sleeping. Estimate the mass of air you exhale. Why are these values different? How much mass loss will you experience during a typical sleep due to breathing alone? Again, state all assumptions clearly and show your logic.

Solution : We begin by making some estimates (consulting appropriate data sources) regarding the number of breaths a person takes per minute, and the volume of inhaled air per breath. Typcial values you will encounter are 16 breaths/minute and 0.75 L per breath. This means that during the course of a night (of 8 hours), an individual take 7680 breaths :

$$16 \text{ breaths} / \text{min x } 60 \text{ min} / \text{hr x } 8 \text{ hr} = 7680 \text{ breaths}$$

Now, being told that each breath inhales a mass of 1 g of air, this means a person inhales 7680 g of air during the night. But how much air do you exhale? We assume that the volume of an exhale is equal to the volume of an inhale, and recall that the gas exhaled is carbon dioxide, whose mean molecular weight of 44 is greater than the mean molecular weight of air (29). Thus, if each inhale of air has a mass of 1 g, then each exhale has a mass that is 44/29 greater, or about 1.5 g. This means that each inhale/exhale results in the loss of 0.5 g, or 3840 g (3.84 kg) during the night. This is too large by about a factor of 6. Some possible explanations are that each exhale is not pure carbon dioxide, but contains a significant amount (approx 20 %) of air. Also, the body does not absorb all the air (or the oxygen in the air) during each breath or possibly I am using too high an estimate for the volume of each breath. But for a simple calculation using a very primitive model of breathing, we get within an order of magnitude, which at least points us in the right direction.

If you wanted to compute the mass of air from first principles, you had at least a couple of ways to do that. Both involve the ideal gas law which I suspect you learned as :

$$P V = n R T$$

where P is the pressure, V is the volume, n is the number of moles, R is the gas constant, and T is the temperature. Physicists, by the way, tend not to use this form and use instead P = N k T where

N is the total number of particles and k is the Boltzmann constant; this is a much better form when dealing with systems that do not have well known volumes. Back to the lung. Using this equation will require a little bit of care in choosing units. If you encountered the ideal gas law in chemistry, you probably used the horrid system of atmospheres, liters, and R was measured in "liter atmospheres/mole degree Kelvin". In physics, we will use the SI MKS system, in which :

P is measured in N/m^2 (called the Pascal, Pa); 1 atm $\approx 10^5\ N/m^2$

V is measured in m^3 (and 1 liter $= 10^{-3}\ m^3$)

T is measured in Kelvin

R, the gas constant, then has a value of 8.31 J / K (J $\equiv$ Joules) $= 8.31\ N \cdot m/^o K$

Then the ideal gas law allows us to compute the number of moles in each breath :

$$n = \frac{P\,V}{R\,T} = \frac{10^5\ N/m^2 \cdot 0.5 \cdot 10^{-3}\ m^3}{8.31\ N\,m/K \cdot 300\ K} = 0.03\ \text{mole}$$

Since each mole of air has a mass of 29 grams, this yields a mass of approx 1 g per breath.

Alternately, you could have estimated the number of moles per breath by recalling the result that at STP, 1 mole of gas occupies 22.4 L; thus, a breath of 0.75 L would represent 0.75 L/22.4 L/mole = 0.03 moles of gas.

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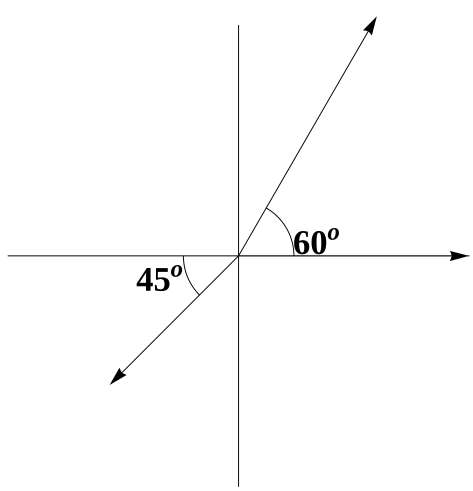
3. A particle undergoes three successive displacements : a) 4.0 meters southwest, b) 5.0 meters east, c) 6.0 meters in a direction 60 degrees north of east. (Choose your coordinates such that the positive y axis represents due north and the positive x axis represents due east). Find :
   a) The components of each displacement
   b) The components of the resultant displacement
   c) The magnitude and direction of the resultant displacement.

Solution : We consider the following diagram :



which shows the three vectors and the angles they make with the coordinate axes. Our first task is to compute components :

$A_x = -4\cos 45 = -2.8$ $\qquad$ $A_y = -4\sin 45 = -2.8$

$B_x = 5$ $\qquad$ $B_y = 0$

$C_x = 6\cos 60 = 3$ $\qquad$ $C_y = 6\sin 60 = 5.2$

The resutant is the vector sum **R** = **A** + **B** + **C**, so that :

$$R_x = A_x + B_x + C_x = -2.8 + 5 + 3 = 5.2$$
$$R_y = A_y + B_y + C_y = -2.8 + 5.2 = 2.4$$

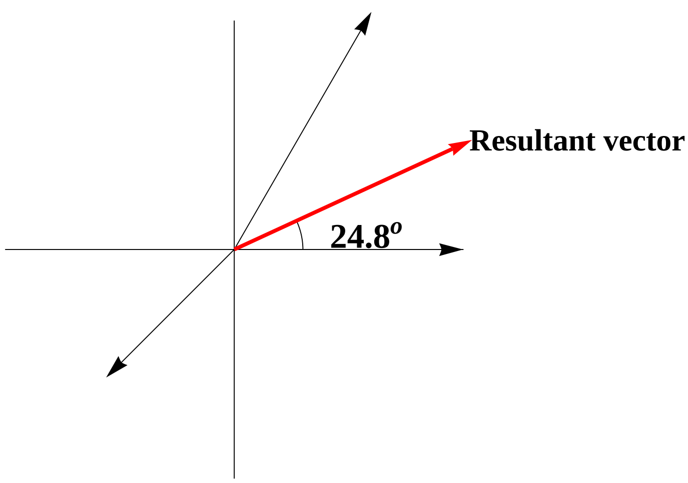
The magnitude of the vector R is :

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{5.2^2 + 2.4^2} = 5.7$$

The direction of the vector R is given by :

$$\tan\theta = \frac{R_y}{R_x} = \frac{2.4}{5.2} \Rightarrow \theta = \tan^{-1}\left(\frac{2.4}{5.2}\right) = 24.8^{o}$$

The diagram below shows the initial vectors with the resultant included in red.



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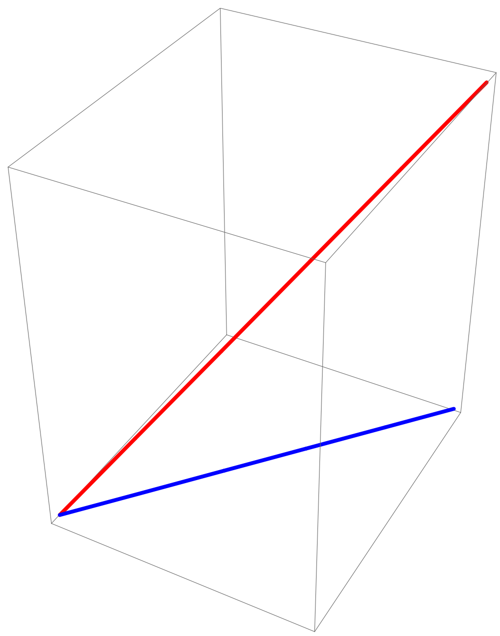
4. A room has dimensions of 10 ft x 12 ft x 14 ft. A fly starting at one corner ends up at the diametrically opposite corner.   What is the magnitude of the displacement?

Solution : Consider the diagram below showing our room of 10 x12x14 dimensions. The red line represents the vector from one corner to the diametrically opposite; the blue line represents the the vector across the diagonal of the floor of the room.



The total displacement of the fly will be the length of the red vector, given by the Pythagorean theorem as :

$$\text{displacement} = \sqrt{(10\text{ ft})^2 + (12\text{ ft})^2 + (14\text{ ft})^2} = 21\text{ ft}$$

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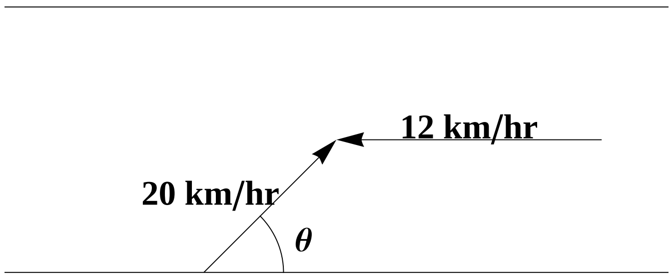
5. A boat can travel with a speed of 20 km/h in still water. If the boat is to travel directly across a river whose current has a speed of 12 km/hr, at what upstream angle must the boat head? Assume the boat needs to travel from the south to the north, and the current flows exactly from the east to the west.

Solution : In thinking about this situation, we should realize that if the boat tries to travel south to north without taking the current into account, the boat' s total motion will consist of both a south to north component and an east to west component. The statement of the problem shows that we wish the boat to have only a south to north component. This means that the boat needs to angle into the current in such a way to ensure that its resultant motion vector has only a south - north component; in other words, we must choose the angle of travel such that the total east - west component is zero. The diagram below illustrates the current of 12 km/hr flowing to the west, and the boat entering the water at an angle θ with respect to the southern shore. In order to have motion only in the south - north direction, the resultant east - west component must be zero. Therefore, we have that 20 cos θ = 12 or $\theta = \cos^{-1}\left(\frac{12}{20}\right) = 53^o$. The boat must angle into the water at an angle of 53^o(or an angle of 37^o with its intended direction of motion).


12 km/hr
20 km/hr
θ