PHYS 111
HOMEWORK #5

Solutions

Submit homework in proper format. Solutions must be accompanied by complete work and/or explanations to receive full credit. In all physics problems, but especially those involving forces, it is very useful to draw free body diagrams representing all the forces.

1. Use the scenario from problem 6 of the last homework set, except now the projectile is launched downward with an initial velocity of 30 m/s at an angle 40 degrees below the horizontal. Compute time of flight, the horizontal distance from the edge of the cliff the object lands, the x and y components of its final velocity, and the magnitude and direction of the final velocity vector. Compare this velocity vector to the one we obtain for problem 6 on the previous homework; what relationship do they have to each other? Explain why they bear this relationship. (25 points total; 5 pts for each part)

Solutions : The difference between this problem and problem 6 of the last assignment is that in this case, the initial velocity is directed downward. Setting our coordinates so that up is the positive direction, the equation of motion in the y direction becomes :

$$y\,(t) \;=\; H \;-v_o\sin\theta\, t \;-\; \frac{1}{2}\, g\, t^2$$

Substituting values :

$$y\,(t) \;=\; 100 \;-\; 30\sin 40\, t \;-\; \frac{1}{2}\, g\, t^2$$

To find the total time of flight, we solve the equation when y (t) = 0 :

$$0 = 100 - \; 30\sin 40\, t \;-\; 4.9\, t^2$$

Which yields the quadratic :

$$t \;=\; \frac{30\sin 40 \;\pm\; \sqrt{(-30\sin 40)^2 - 4\,(-4.9)\,(100)}}{-9.8} \;=\; \frac{19.3 \pm \sqrt{19.3^2 + 1960}}{-9.8} \;= 2.96\text{ s}$$

Since there are no forces in the x direction, the horizontal component of motion, 30 cos 40 = 23 m/s, is constant, so the total horizontal distance traveled is :

$$\text{Range} \;=\; v_x\,(\text{time of flight}) \;=\; 23\text{ m/s}\cdot 2.96\text{ s} \;=\; 68\text{ m}$$

The x component of velocity has just been determined to be 23 m/s; the final y component is found from :

$$v_y\,(t) \;=$$

$$v_{oy} + at = v_{oy} - g\,t = -v_o \sin\theta - g\,t = -30 \sin 40\,m/s - 9.8\,\frac{m}{s^2} \cdot 2.96\,s = -48.3\,m/s$$

Notice that these are exactly the same components as we found in problem 6 of the last homework (so that the final velocity vector will be identical the one for that problem.) This might seem unexpected, but let' s consider why the final velocity vector is the same in both cases. Recall that the final velocity vector is the resultant of the velocity components in the x and y direction. We already know that the x component will be the same (no forces in the x direction), so we focus on the final y velocity. We can compute the final y velocity from the equation :

$$v_{f\,y}^2 = v_{o\,y}^2 + 2\,a\,\Delta y$$

where $v_{f\,y}$ is the final velocity in the y direction, $v_{o\,y}$ is the initial velocity in the y direction, a is the acceleration and Δy is the displacement in the y direction. Here, the initial velocity is $\pm v_o \sin\theta$ (depending on whether the projectile is launched upward or downward); $a = -g$, and the displacement in the y direction is $y_{final} - y_{initial} = 0 - H = -H$. Thus :

$$v_{f\,y}^2 = (\pm\, v_o \sin\theta)^2 + 2\,(-g)\,(-H) = v_o^2 \sin^2\theta + 2\,g\,H$$

and the final velocity has the same magnitude whether we launched up or down, assuming the magnitude of velocity and angle with respect to the horizon are the same.

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2. Problem 34, text, p. 96. Be careful to convert everything to a consistent set of units. 1 g = 9.8 m/s^2

Solution : We find the linear speed by recognizing that a point on the tip of the blade is moving in a circular path of radius 3.4 m. Therefore, each revolution has a circumference of

$$C = 2\,\pi\,r$$

where r is the radius (3.4 m) of the path. If the blade completes 550 revolutions per minute, then the linear speed of a point on the blade tip is :

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{550\,\frac{\text{revs}}{\text{min}} \cdot 2\,\pi\left(3.4\,\frac{m}{\text{rev}}\right)}{1\,\text{min} \cdot 60\,\frac{s}{\text{min}}} = \frac{11\,743\,m}{60\,s} = 196\,\frac{m}{s}$$

The radial acceleration is found from :

$$a_{radial} = \frac{v^2}{r} = \frac{(196\,m/s)^2}{3.4\,m} = 11\,267\,\frac{m}{s^2}$$

Compared to the acceleration of gravity, this is :

$$a = 11\,267\,\frac{m}{s^2} \Big/ \left(9.8\,\frac{m}{s^2} \Big/ g\right) = 1150\,g's$$

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3. Problem 58, text, p. 97.

Solution : The bird' s motion has two components : one in the horizontal plane and one in the vertical plane. In the horizontal plane, we use the techniques of the previous problem to determine that the linear speed in the horizontal plane is :

$$\text{speed} = \frac{2\pi\,(8\text{ m})}{5\text{ s}} = 10\text{ m/s in the horizontal plane.}$$

The total speed is found from the Pythagorean Theorem :

$$\text{total speed} = \sqrt{(\text{horizontal speed})^2 + (\text{vertical speed})^2} = \sqrt{(10\text{ m/s})^2 + (3\text{ m/s})^2} = 10.4\text{ m/s}$$

b) The only acceleration is due to the radial component, so the acceleration is :

$$a_{\text{radial}} =$$

$$\frac{v_{\text{plane}}^2}{r} = \frac{(10\text{ m/s})^2}{8\text{ m}} = 12.5\,\frac{\text{m}}{\text{s}^2}\ \text{directed toward the center of the circular path executed by the bird.}$$

c) One component of the velocity lies in a horizontal plane; the other is perpendicular to the horizontal (in the vertical plane). Therefore, with respect to the horizontal we have :

$$\tan\theta = \frac{\text{vertical velocity}}{\text{horizontal velocity}} = \frac{3\text{ m/s}}{10\text{ m/s}} \Rightarrow \theta = \tan^{-1}(0.3) = 16.7\text{ degrees.}$$

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4. Problem 62, text, p. 97

Solution : For this problem, I will use a coordinate system where the landing point (opposite side of the river) is at y = 0, so that up is positive, and the launch point is at y = H = 15 m. The equations of motion are then :

$$x(t) = v_o \cos\theta\, t$$

$$y(t) = H + v_o \sin\theta\, t - \frac{1}{2} g t^2$$

Where are asked to find the launch velocity that will allow the motorcycle to just land on the edge of the opposite bank so that we know that x = 40 m when y = 0. As done in the text and book, we solve for t and substitute this into the y equation of motion (this gives us an equation for y in terms of x) :

$$t = \frac{x}{v_o \cos\theta}$$

$$y(x) = H + v_o \sin\theta\left(\frac{x}{v_o\cos\theta}\right) - \frac{g x^2}{2 v_o^2 \cos^2\theta} = H + x\tan\theta - \frac{g x^2}{2 v_0^2 \cos^2\theta} \tag{1}$$

When y = 0, x = 40; substituting the given values we have :

$$0 = 15\text{ m} + 40\tan 53\text{ m} - \frac{\left(9.8\text{ m}/\text{s}^2\right)(40\text{ m})^2}{2 v_0^2 (\cos 53)^2}$$

which yields :

$$0 = 68\,\text{m} - \frac{\left(21\,646\,\text{m}^3 / \text{s}^2\right)}{v_o^2} \Rightarrow v_o = \sqrt{\frac{21\,646\,\text{m}^3 / \text{s}^2}{68\,\text{m}}} = 17.8\,\text{m}/\text{s}$$

(You can check this result by subsituting this value for initial velocity into equation (1)). If the initial velocity were half this value, we can determine whether the bike even reaches the other side and crashes into the side of the cliff by finding the value of y when x = 40 m :

$$y\,(40) = 15\,\text{m} + (40 \tan 53)\,\text{m} - \frac{\left(9.81\,\text{m} / \text{s}^2\right)(40\,\text{m})^2}{2\,(8.9\,\text{m}/\text{s})^2\,(\cos 53)^2} = -205\,\text{m}$$

This shows us that the bike has hit the ground (at a height of - 85 m) long before it reaches the other side (40 m away). So, we go back to our equation of motion and find the value of x when y = -100. We have :

$$-85\,\text{m} = 15\,\text{m} + x \tan 53 - \frac{g\,x^2}{2\,v_o^2 \cos^2 53}$$

Which yields this quadratic in x :

$$-100 = 1.3\,x - 0.17\,x^2$$

which has the realistic solution x = 28.4 m. In other words, at half the minimum launch velocity, the bike crashes into the river at a horizontal distance 28 m from the launch site.

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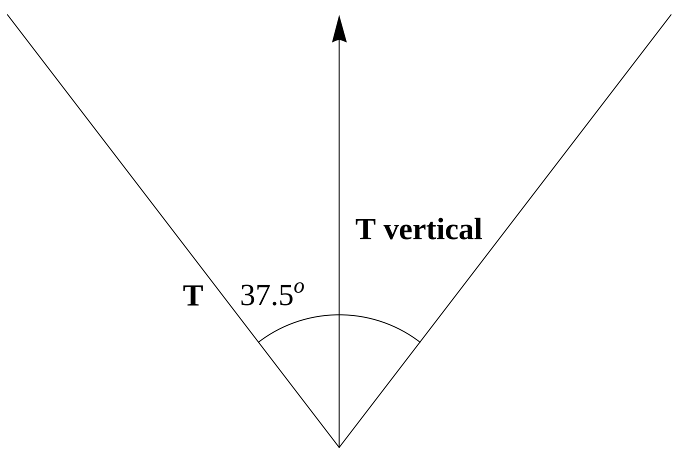
5. Problem 4, text, p. 124

Solution : We are told the tension is the same throughout the strap. We can conceptualize this as a left hand strap and a right hand strap, each of which have the same tension. We also know that we want the total vertical component of the tension to be 5 N. In other words, we want a vertical component of force of 2.5 N on each side of the strap. We can consider the diagram :



We want to find the tension T such that the vertical component of tension on each side of the strap will be 2.5 N, in other words we want to find T such that :

$$\cos 37.5 = \frac{T_{\text{vertical}}}{T} \Rightarrow T = \frac{T_{\text{vertical}}}{\cos 37.5} = \frac{2.5\text{ N}}{\cos 37.5} = 3.1\text{ N}$$

If the tension in the strap is 3.1 N, each side will contribute a vertical component of 2.5 N, for a total vertical component of tension equal to 5 N

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6. Use the diagram for problem 16 on p. 123. Two boxes are in contact on a frictionless table. If the applied force is 3 N, and the mass of block A is 2 kg and the mass of B is 1 kg, find the acceleration of the combined system, AB. Find the contact force between the two blocks. If the force of 3 N is applied to block B, show that the contact force between the blocks is 2 N (which is not the same force as derived before)

Solution : The mass of the combined system is 3 kg. Therefore, if subjected to an applied force of 3 N, F = ma yields that the acceleration of the total system is 1 m/s/s. Now, we apply the second law to block B only, and we have:

$$\text{sum of forces on B} = \text{mass of B} \cdot \text{acceleration of B}$$

Since the system moves as a unit, we have already calculated the acceleration of B (1 m/s/s). The only force acting on B (in the horizontal direction) is the contact force exerted by A, so we have :

$$\text{contact force between A and B} = \text{mass of B} \cdot 1\text{ m/s/s}$$
$$\text{contact force} = 1\text{ kg} \cdot 1\text{ m/s/s} = 1\text{ N}$$

Notice that we can now apply the second law to block A only; there are two horizontal forces acting on A : the applied force of 3 N and the equal and opposite contact force due to B. For block A, we have :

$$\text{sum of forces on A} = 3\text{ N} + (-1\text{ N}) = \text{mass of A} \cdot \text{acceleration of A}$$
$$2\text{ N} = 2\text{ kg} \cdot 1\text{ m/s/s}$$

which confirms our analysis. Now, consider the applied force acting on B. Here, we first find the contact force of B on A. The only force A experiences directly is this contact force, so we have :

contact force on A due to B = mass of A · 1 m/s/s = 2 N. The third law tells us that if this is the contact force of B on A, this must also be the contact force of A on B.

7. Problem 52, text, p. 127

Solution : Refer to the example on p. 120. If you weigh 683 N in a non accelerating reference frame, you know your mass = W/g = 69.7 kg.

Now, if you are in an elevator accelerating up, the forces acting on you are the normal force of the floor on you (acts up), and gravity (your weight acting down). Newton' s second law applied to you is :

$$\text{sum of forces on you} = N - m\,g = m\,a$$

Now, the normal force of the scale on you is equal (by virtue of the third law) to the force you exert on the scale. Thus, the reading on the scale tells you the magnitude of the normal force acting on you. Therefore, we have :

$$725\text{ N} - 683\text{ N} = 69.7\,a \Rightarrow a = 0.6\text{ m}/\text{s}^2$$

In the case where you are accelerating downward :

$$N - mg = ma$$

$$595\,\text{N} - 683\,\text{N} = 69.7\,a \Rightarrow a = -1.26\,\text{m}/\text{s}^2$$