PHYS 111
HOMEWORK #6

Solutions

All problems from the text.

1. Problem 4, p. 152

Solution : We apply Newton' s second Law to the 72 N and 150 N weight. The forces acting on the 150 N weight are the tension in string A (which acts up on this weight), and the force of gravity (the weight) acting down. Thus, for the 150 N object, we have :

$$\Sigma F = m\,a \Rightarrow T_A - 150\,\mathrm{N} = 0 \Rightarrow T_A = 150\,\mathrm{N}$$

It is important to note that the 72 N weight and the upper strings do not enter into the second law equation for the 150 N object. Only those forces which act directly on it are relevant.

Now, the forces acting on the 72 N weight are the tensions in strings B and C, the weight of the object, and the tension in string A which pulls down on the 72 N object. Since the system is in equilibrium, we know that all forces in both the x and y directions balance, so we can write for the vertical forces acting on the weight :

$$\Sigma F = m\,a = T_B \sin 60 + T_C \sin 60 - T_A - 72\,\mathrm{N} = 0$$

Notice that the vertical components of tension in B and C both pull up, and we must include the downward pull of string A as well as its weight. We know from above that the tension in A is 150 N, so the vertical force balance gives us :

$$T_B \sin 60 + T_C \sin 60 = 222\,\mathrm{N} \tag{1}$$

This equation has two unknowns (the tensions in the two strings), so we need one more independent equation. We obtain this from horizontal equilibrium; the horizontal components of the tensions in B and C must sum to zero, so we have :

$$T_B \cos 60 - T_C \cos 60 = 0 \Rightarrow T_B = T_C$$

Now, we could have made this observation at the beginning since we recognize the strings make the same angle with the horizontal, but I wanted to show you the full analysis that will work in all examples of these types of problems. Now, we can return to equation (1) and find that :

$$2\,T_B \sin 60 = 222\,\mathrm{N} \Rightarrow T_B = 128\,\mathrm{N}.$$

This is equal to the tension in string C.

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2. Problem 12, p. 153

Solution : This is similar to the last problem; we should notice right away that the two strings make different angles with respect to the horizontal, so that we should expect different tensions in them. As always, we apply Newton' s second law and find all the forces acting on the man in the x direction and separately in the y direction. Since the system is in equilibrium, we know that :

$$\Sigma F_x = 0 = \Sigma F_y$$

The only forces acting in the x direction are the horizontal components of tension; calling forces to the right positive and forces to the left negative we get :

$$T_R \cos 48 + (-T_L) \cos 35 = 0 \Rightarrow T_R \cos 48 = T_L \cos 35 \Rightarrow T_R = \frac{\cos 35}{\cos 48} T_L = 1.22\, T_L$$

In the y direction, the vertical components of tension must sum to equal the man' s weight, so we have :

$$T_L \sin 35 + T_R \sin 48 = 73\,\text{kg} \cdot 9/8 \frac{\text{m}}{\text{s}^2} = 715\,\text{N}$$

Substituting :

$$T_L \sin 35 + 1.22\, T_L \sin 48 = 715\,\text{N}$$

$$0.57\, T_L + 0.91\, T_L = 1.48\, T_L = 715\,\text{N} \Rightarrow T_L = 483\,\text{N}$$

Finally, since $T_R = 1.22\, T_L$, $T_R = 589$N

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3. Problem 24, p. 153

Solution : For this problem, we apply Newton' s second law to the top of the chain. Tension (acting upward) must support the combined weight of the chain and the boulder (see example 4.8 on p. 119 for a complete explanation of this). Thus, Newton' s second law applied to the top of the chain gives :

$$\Sigma F = \text{Tension in chain} - \text{weight of chain} - \text{weight of rock} = (\text{total mass}) \cdot a$$

We are also told that the tension in the chain cannot exceed 2.5 times the weight of the chain. If we apply the maximum tension to the chain, we will produce the greatest upward acceleration of the system. We can write the second law in this case as :

$$2.5\, W_{chain} - W_{chain} - W_{rock} = (m_{chain} + m_{rock})\, a_{max}$$

or

$$1.5\, W_{chain} - W_{rock} = (m_{chain} + m_{rock})\, a_{max}$$

Solving for the maximum acceleration :

$$a_{max} = \frac{1.5\, W_{chain} - W_{rock}}{m_{chain} + m_{rock}}$$

Now, substituting numerical values we obtain :

$$a_{max} = \frac{1.5\,(575 \cdot 9.8)\,\text{N} - 750 \cdot 9.8\,\text{N}}{575\,\text{kg} + 750\,\text{kg}} = 0.83\,\text{m}/\text{s}^2$$

To find the time it takes to lift this from a 125 m quarry, use the equation of motion :

$$y(t) = y_o + v_o t + \frac{1}{2} a t^2$$

If we set the bottom of the quarry to y = 0, we know the initial velocity is zero. These initial conditions give us :

$$125\,\text{m} = \frac{1}{2} * 0.83\,\frac{\text{m}}{\text{s}^2}\,t^2 \Rightarrow t = 17.3\,\text{s}$$

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4. Problem 28, p. 154

Solution : This problem is very similar to example 5.7 on pp. 136 - 137. We apply Newton' s second law to each bucket. For the 22 kg bucket, we set down as the positive direction, so we have :

$$W\,(22\,\text{kg bucket}) - T = m\,a \tag{2}$$

where W is the weight of the 22 kg bucket, T is the tension in the string, m is the mass of the 22 kg bucket (so we can also write the weight of this bucket as m g) , and a is the acceleration of thesys-tem. There is no friction acting on the roof, so the only force acting on the 375 N box is the tension in the string. The tension is the same everywhere in the string, so adopting a system where positive is to the right, we have :

$$T = M\,a$$

where M is the mass of the box. Using this expression for M in equation (2), we get (and setting W = m g) :

$$m\,g - M\,a = m\,a \Rightarrow m\,g = m\,a + M\,a$$

or

$$a = \frac{m\,g}{m + M} = \frac{22\,\text{kg} \cdot 9.8\,\text{m}/\text{s}^2}{22\,\text{kg} + \left(375\,\text{N}/9.8\,\text{m}/\text{s}^2\right)} = 3.56\,\text{m}/\text{s}^2$$

If the bucket started from rest, we use the kinematic equation :

$$v_f^2 = v_o^2 + 2\,a\,x$$

where v_f = final velocity

v_0 = initial velocity = 0

a = acceleration = $3.56\,\text{m}/\text{s}^2$

x = displacement = 1.5 m

and find :

$$v_f^2 = 0 + 2 \cdot 3.56\,\frac{\text{m}}{\text{s}^2} \cdot 1.5\,\text{m} \Rightarrow v_f = 3.27\,\text{m}/\text{s}$$

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5. Problem 34, p. 154

Solution : We apply Newton' s second law to each box. The forces acting on box B are the pulling force, F, to the right, friction to the left, and the tension in the string to the left; vertically, we have the weight (down) and the normal force (up). We are also told that the box is moving with constant speed, this means that it is not accelerating. This means that there are no net forces acting on the box. We have two relevant equationsfor box B :

$$\Sigma F_x = F - f_B - T = 0 \quad \text{and}\, \Sigma F_y = W_B - N_B = 0 \tag{3}$$

We see that the normal force equals the weight of B, so that we can determine the frictional force acting on B :

$$f_B = \mu_K N_B = \mu_k m_B g$$

The horizontal forces acting on box A are the tension in the string and friction; the vertical forces are the weight of A and the normal force. Newton' s second law gives :

$$\Sigma F_x = T - f_A = 0 \qquad \Sigma F_y = N_A - W_A = 0$$

The normal force here equals the weight of A, so that the frictional force is

$$f_A = \mu_k m_A g$$

The x equation for box A shows that the tension equals the frictional force on A, therefore, the x equation in equation (3) becomes :

$$\Sigma F_x = F - f_B - T = F - \mu_k m_B g - \mu_k m_A g = 0$$

$$F = \mu_k (m_A + m_B) g$$

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6. Problem 42, p. 154

Solution : For this problem, see figs. 5.16 and 5.17 on p. 143 for a diagram of the situation and a free body diagram. We sum forces along the plane of the incline. There is a component of gravity acting down the plane with magnitude m g sin θ, and if there is friction, it acts up the plane with a magnitude given by μ m g cos θ where μ is the coefficient of friction. In the case of no friction, Newton' s second law along the plane gives us :

$$F = m a = m g \sin\theta \Rightarrow a = g \sin\theta = 9.8\, \mathrm{m}/\mathrm{s}^2 \sin 40 = 6.3\, \mathrm{m}/\mathrm{s}^2$$

In the case of friction, we have :

$$\Sigma F = m g \sin\theta - \mu m g \cos\theta =$$
$$m a \Rightarrow a = g (\sin\theta - \mu \cos\theta) = 9.8\, \mathrm{m}/\mathrm{s}^2 (\sin 40 - 0.2 \cos 40) = 4.8\, \mathrm{m}/\mathrm{s}^2$$

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7. Problem 48, p. 154

Solution : This is similar to the problem done in class. The vertical forces acting on the box are its weight (down), the normal force (up), and the vertical component of the pushing force (call it P) which is down in parts b - d, and acts up in part e; horiztonally, there is the horizontal component of P and the frictional force. If the box is not moving, we know it is in equilibrium and sum of forces in each direction is zero. We write Newton' s second law :

$$\Sigma F_x = P_{hor} - f = 0$$

$$\Sigma F_y = N - W - P_{vert} = 0$$

We know also that :

$$f = \mu_s N$$

We obtain :

$$P \cos\theta - \mu_S N = 0 \tag{4}$$

$$N = W + P \sin\theta \tag{5}$$

Substituting the given values into eq. (5) yields :

$$N = 125\,N + 75\,N \sin 30 = 162\,N$$

and the frictional force is f = 0.8*162 N = 130 N

The maximum frictional force occurs if P directs straight down. In this case the normal force is 125 N + 75 N = 200 N, and the frictional force is μ N = 160 N

If P is directed upward at 30 degrees, Newton' s second law in the vertical direction becomes :

$$N - W + P \sin 30 = 0 \Rightarrow N = W - P \sin 30 = 125\,N - 37.5\,N = 88\,N$$

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8. Problem 58, p. 154

Solution : The graph of F vs. x is a straight line indicating that we can write the relationship between F and x as :

$$F = k\,x + b$$

where k is the slope (and here is the spring constant) and b is the y - intercept, which you can read from the graph is zero. Therefore, this graph does in fact represent Hooke' s law, and the slope of the straight line is the spring constant. The measured force is 15 N when the spring is displaced 10 cm (0.1 m), so we can determine the value of k :

$$k = \frac{F}{x} = \frac{15\,N}{0.1\,m} = 150\,N/m$$

Using this value for the spring constant, it would take a force of :

$$F = 150\,N/m * 0.17\,m = 25.5\,N$$

to stretch the spring 17 cm (0.17 m) from its equilibrium position.

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9. Problem 72, p. 158

Solution : The key to this problem is recognizing that the acceleration of the elevator increases the normal force acting on the box. In the vertical direction, Newton' s second law gives :

$$\Sigma F_y = N - W = m\,a$$

Here, the right hand side is non - zero since the box is accelerating upward. Therefore, the normal force is :

$$N = W + m\,a = m\,(g + a)$$

If the box slides with a constant velocity, the horizontal forces on the box sum to zero, meaning that the force you must apply equals the force of kinetic friction. We have :

$$\text{Force you apply} = \mu_K\,N = \mu_K\,m\,(g + a) = 0.32 * 28\,kg\,(9.8 + 1.9)\,m/s^2 = 105\,N$$

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10. Problem, 87 p. 159

Solution : Recall that the magnitude of friction depends on the coefficient of friction between the surfaces, and the force pressing the surfaces together. Since the cart is accelerating to the right, this causes the block to accelerate, and this acceleration generates a force on the block. This is the source of the normal force between the two surfaces. (The weight of the block does not exert a force on the block since the direction of weight is straight down.). Thus, the normal force acting on the block is then simply M a, where M is the mass of the block and a the acceleration of the system. (It was not wise of the book to use A to represent the block, so don' t be confused by that "A"). Now, the vertical forces on the block are its weight, M g, and the frictional force acting upward, equal to μ N = μ M a. In order for the block NOT to accelerate downward, the frictional force acting up must equal the weight of the block, or :

$$\mu \, M \, a = M \, g \Rightarrow a = \frac{g}{\mu}$$