PHYS 111

HOMEWORK #7

Solutions

All problems from the text. All questions must be answered completely, explaining and/or showing all work.

Conceptual Questions :

1. No. 2, p. 182.

Answer : If the car is rounding a curve at constant speed, we know there is a net centripetal force acting toward the center of the circle. If the car is not accelerating up or down the plane, the net force acts radially inward.

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2. No. 8, p. 182

Answer : The reason cited in the book is clearly false; of course there is gravity acting on the astronauts, that is the force keeping them in orbit around the Earth. The astronauts are weightless because they are falling toward the center of the Earth; they have sufficient forward (tangential) velocity to keep from spiraling toward the surface. The astronauts are weightless since there are no reaction forces acting on them.

Problems

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3. Problem 4, p. 183

Solution : As we have shown in class, the second law applied to the vertical direction gives :

$$\Sigma F_y = N - W = 0 \Rightarrow N = W = m g.$$

Horizontally, the frictional force produces the centripetal acceleration, or :

$$\Sigma F_x = f = \mu N = m v^2 / r$$

$$\mu m g = m v^2 / r \Rightarrow \mu = \frac{v^2}{r g}$$

Substituting values,

$$\mu = (25\,m/s)^2 / \left(220\,m \cdot 9.8\,m/s^2\right) = 0.29$$

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4. Problem 10, p. 183

Solution : At the lowest point in its orbit, the forces acting on the rope are the tension (acting up), its weight (acting down), and the combination of these forces produces a centripetal acceleration (of magnitude v^2 / r) acting up toward the pivot point. Newton' s second law gives :

$$T - m g = \frac{m v^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right) = 7.27\,\text{kg}\left(9.8\,\text{m}/\text{s}^2 + (4.2\,\text{m}/\text{s})^2/3.8\,\text{m}\right) = 105\,\text{N}$$

Note that I used W = m g to compute the mass of the object given its weight.

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5. Problem 13, p. 183

Solution : The information given in the problem allows us to compute the velocity of the swinging arm. We are told that the arm swings in a circular arc of radius 0.7 m, and completes 1/8 of a revolution in 0.5 s. Recalling that a full revolution has a circumference of $2\pi r$, the velocity of the swinging arm is :

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r\left(\frac{1}{8}\right)}{t} = \frac{2\pi(0.7\,\text{m})\left(\frac{1}{8}\right)}{0.5\,\text{s}} = 1.10\,\text{m}/\text{s}$$

The acceleration of a drop of blood at the tip of the is the centripetal acceleration :

$$a_{cent} = \frac{v^2}{r} = \frac{(1.10\,\text{m}/\text{s})^2}{0.7\,\text{m}} = 1.73\,\text{m}/\text{s}^2$$

We can use the result of the previous problem to determine the force acting on the droplet (whose mass is 0.001 kg) :

$$F = m\left(g + \frac{v^2}{r}\right) = 0.001\,\text{kg}\,(9.8 + 1.73)\,\text{m}/\text{s}^2 = 0.0115\,\text{N}$$

If the arm were hanging straight down without swinging, then there would be no acceleration and the sum of forces acting on the droplet would be zero. The forces would be the capillary force exerted by the blood vessel and the droplet' s weight, or :

$$F = m\,g = 0.001\,\text{kg} * 9.8\,\text{m}/\text{s}^2 = 0.0098\,\text{N}$$

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6. Problem 18, p. 184

Solution : First, we realize that each astronaut exerts a gravitational force on the other. The magnitude of this force on each astronaut is :

$$F_{grav} = \frac{G\,m_1\,m_2}{r^2}$$

where m_1 and m_2 are the masses of the two astronauts. Since the gravitational force is the net force acting on each astronaut, we can find the acceleration of each astronaut by setting :

$$F(\text{on } m_1) = G\frac{G\,m_1\,m_2}{r^2} = m_1\,a_1 \Rightarrow a_1 = \frac{G\,m_2}{r^2}$$

We can apply the same reasoning to the second mass and find that :

$$a_2 = \frac{G\,m_1}{r^2}$$

Substituting values, we find :

$$a_1 = \left(6.67\times 10^{-11}\ \text{N m}^2\ \text{kg}^2\right) * 72\ \text{kg} / (20\ \text{m})^2 = 1.2\times 10^{-11}\ \text{m}/\text{s}^2$$
$$a_2 = \left(6.67\times 10^{-11}\ \text{N m}^2\ \text{kg}^2\right) * 65\ \text{kg} / (20\ \text{m})^2 = 1.08\times 10^{-11}\ \text{m}/\text{s}^2$$

These are the initial accelerations acting on the astronauts. As they approach each other, their acceleration will increase since their separation will decrease. To compute a more precise time of collision, we would have to use elements of calculus to compute this time; since we cannot assume calculus in this course, the book asks you to make the (incorrect) assumption that the astronauts will move toward each other with these constant accelerations. If they start from rest, we know the distance each will travel in a time t will be :

$$s = \frac{1}{2} a t^2$$

Since they collide after a time t, we want to find the time such that

$$s_1 + s_2 = 20\ \text{m} = \frac{1}{2}(a_1 + a_2) t^2$$

or :

$$t = \sqrt{\frac{2 * 20\ \text{m}}{(1.20 + 1.08)\ 10^{-11}\ \text{m}/\text{s}^2}} = 1.32\times 10^6\ \text{s} = 15.3\ \text{days}$$

Gravity is truly a weak force.

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7. Problem 24, p. 185

Solution : a) Masses B and C each exert a gravitational force on A. Because of the geometry of the situation, we can see that the x component of the force exerted by B on A is equal to and opposite the x component of the force exerted by C on A. Therefore, the net force on A has no x component, and only a y component. If F is the total force of B (or C) on A, then the y component of F is given by F sin 60 (since each angle in an equilateral triangle is 60 degrees). Since both B and C exert this force on A, the total force on A is

$$2\ F \sin 60$$

in the positive y direction, where the magnitude of F is found from :

$$F = \frac{G\, m_1\, m_2}{r^2} = \left(6.67\times 10^{-11}\ \text{N m}^2/\text{kg}^2\right)(3\ \text{kg})\,(3\ \text{kg})/(0.1\ \text{m})^2$$

b) Mass B and C exert forces of equal magnitude on A; the force due to B acts along the positive x axis; the force from C acts along the negative y axis. Since they are of equal magnitude, the resultant of these two forces acts along the diagonal from A to D. Since the force of D on A also acts along this line, we can deduce that the net force acts along the diagonal from A to D.

We will find first the magnitude of the net force from B and C acting on A. Since all the masses are the same, we will just call them m in all cases. We will call r the length of each side. Using these symbols, we can find the magnitude of the net force due to B and C :

$$\text{net force due to B + C} = \sqrt{F_B^2 + F_C^2} = \sqrt{\left(\frac{G\,m^2}{r^2}\right)^2 + \left(\frac{G\,m^2}{r^2}\right)^2} = \frac{G\,m^2}{r^2}\sqrt{2}$$

and this force acts along the diagonal. The force of D on A acts along the diagonal, and the appropriate distance is the length of the diagonal which is given simply by the Pythagorean theorem as :

$$(\text{length of diagonal})^2 = r^2 + r^2 = 2\,r^2$$

Thus, the force of D on A is :

$$F_D = \frac{G\,m^2}{2\,r^2}$$

The total force along the diagonal is then :

$$F_{total} = F_B + F_C + F_D = \left(\sqrt{2} + \frac{1}{2}\right)\frac{G\,m^2}{r^2}$$

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8. Problem 28, p. 185.

Solution : We use proportional reasoning with the definition of weight :

$$W = \frac{G\,M\,m}{R^2}$$

where G is the Newtonian gravitational constant, M is the mass of the planet, m is the mass of the test mass, and R is the radius of the planet. Thus, the new weight, W', is in each case :

a) doubling the mass of the planet :

$$W' = G\,(2\,M)\,m \big/ R^2 = 2\,W$$

b) halving the radius :

$$W' = G\,M\,m \big/ (R/2)^2 = 4\,W$$

c) halving both radius and mass :

$$W' = G\,(M/2)\,m \big/ (R/2)^2 = 2\,W$$

d) doubling both mass and radius :

$$W' = G\,(2\,M) \big/ (2\,R)^2 = W/2$$

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9. Problem 32, p. 185

Solution : Our first task in this problem is to find the surface gravity on 234 Ida :

$$g = \frac{G\,M}{R^2} = \left(6.67\times10^{-11}\ \text{N m}^2 \big/ \text{kg}^2\right)\left(4\times10^{16}\ \text{kg}\right) \Big/ \left(1.6\times10^{4}\ \text{m}\right)^2 = 0.01\ \text{m}/\text{s}^2$$

which is essentially 1/1000 of the surface gravity on the Earth. An astronaut weighing 650 N on the Earth will weigh 1/1000 of this on 234 Ida, or 0.65 N.

We recall from chapter 2 the equation :

$$s = \frac{1}{2} g t^2$$

where s is the distance traveled by an object dropped from rest, g is the local gravitational field, and t is the time of fall. On this asteroid, it would take a time :

$$t = \sqrt{2 s / g} = \sqrt{2 * 1 \text{ m} / 0.01 \text{ m} / \text{s}^2} = 13.8 \text{ s to fall } 1 \text{ m}$$

Recall from chapter 2 that an object launched vertically upward with an initial velocity v reaches a maximum height of :

$$\text{height} = \frac{v^2}{2 g}$$

If the launch velocity is the same, the height achieved will depend on the gravity. If the gravity on 234 Ida is 1000 times smaller on the Earth, an astronaut would reach a height 1000 times great, or a height of 600 m. (This launch speed would not be enough to escape the asteroid, however).

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10. Problem 38, p. 185

Solution : Equating gravitational and centripetal forces gives us :

$$\frac{G M m}{R^2} =$$

$$\frac{m v^2}{R} \Rightarrow v = \sqrt{\frac{G M}{R}} = \sqrt{\left(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^2\right)\left(4 \times 10^{16} \text{ kg}\right) / 1.6\ 10^4 \text{ m}} = 12.9 \text{ m} / \text{s}$$

Compare this to the Earth' s orbital speed of approximately 5 km/s. The time to complete one orbit is :

$$\text{Period} = \frac{\text{circumference of orbit}}{\text{velocity}} = \frac{2 \pi R}{v} = \frac{2 \pi * 1.6 \times 10^4 \text{ m}}{12.9 \text{ m} / \text{s}} = 7789 \text{ s} = 2.16 \text{ hrs}$$

The period for one revolution in low Earth orbit is approximately 88 minutes.

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11. Problem 54, p. 185

Solution :

We apply Newton' s second law to the 4 kg mass. The mass is accelerating in the x direction, but is not accelerating in the y direction. We identify the forces in the y direction as : the vertical component of the tension in the upper string (acts up), the vertical component of tension in the lower string (acts down), and the weight (acts down)

This allows us to write Newton' s second law as :

$$\Sigma F_y = T_u \text{ (vertical)} - T_L \text{ (vertical)} - W = 0$$

where T_u and T_L represent the tensions in the upper and lower strings respectively.

In order to find the components of tension, we need to determine the angle the string makes with the vertical. Using the information given to us in the diagram, we have :

$$\cos\theta = \frac{1}{1.25} = 0.8$$

Now we can solve for the tension in the lower string :

$$T_u \cos\theta - T_L \cos\theta = W$$

$$(T_u - T_L) = \frac{W}{\cos\theta} \Rightarrow T_L = T_u - \frac{W}{\cos\theta}$$

Substituting values :

$$T_L = 80 - \frac{4\,\text{kg} * 9.8\,\text{m}/\text{s}^2}{0.8} = 31\,\text{N}$$

We can find the speed of the block by using Newton' s 2 nd law in the horizontal direction, recognizing that the net horizontal force due to the strings produces a centripetal acceleration :

$$\Sigma F_x = T_u\,(\text{horizontal}) + T_L\,(\text{horizontal}) = \frac{m v^2}{r}$$

We use the Pythagorean theorem to find the value of r :

$$r = \sqrt{\left(1.25\,\text{m}^2\right) - (1\,\text{m})^2} = 0.75\,\text{m}$$

from which we determine that sin θ = 0.75/1.25 = 0.6

Taking the horizontal components of tension gives us :

$$(T_u + T_L)\sin\theta = \frac{m v^2}{r} \Rightarrow v = \sqrt{r\,(T_u + T_L)\sin\theta / m} =$$

$$\sqrt{0.75\,\text{m}\,(80\,\text{N} + 31\,\text{N}) * 0.6 / 4\,\text{kg}} = 3.53\,\text{m}/\text{s}$$