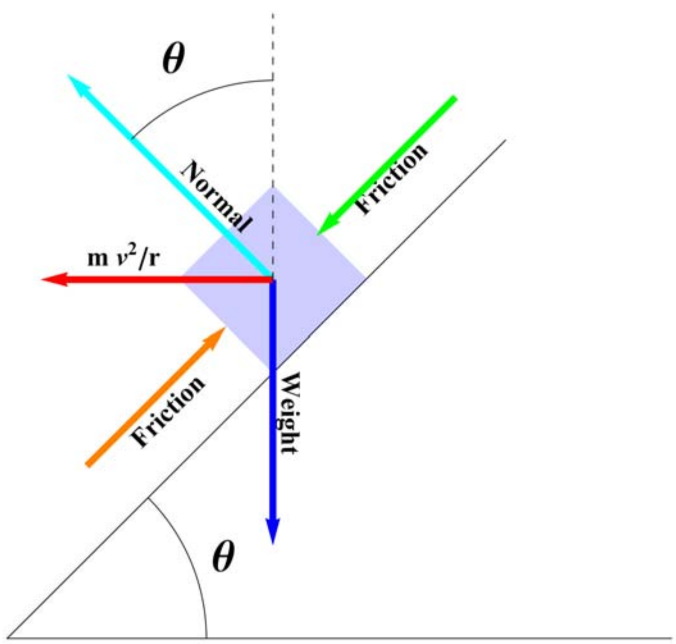
PHYS 111
HOMEWORK #8

Solutions

There has been a trend toward merely asserting or quoting results without showing full work. No work = no credit. All answers must show complete derivations of your results.

1. A car travels along a banked curve of radius R making an angle θ with the horizon. In this case, the coefficient of friction between the car and road is μ. Find an expression for the maximum speed of the car before it slides up the bank. Find an expression for the minimum speed of the car before it slides down the bank.

Solution : We begin by drawing a force diagram to represent this system :



We use the diagram above to consider the two cases of interest here : finding the maximum speed the car can have before it slides up the bank, and then the lowest speed possible before the car slides down the bank. Let' s start by finding the maximum speed.

If we want to keep the car from sliding up the bank, the force of friction will act down the bank (as indicated by the green arrow). Adopting a standard Cartesian coordinate system (the x axis lies along the ground; the y axis lies in the direction of gravity) we find components of forces and write Newton' s second law in the x and y directions :

$$\Sigma F_x = N \sin\theta + f \cos\theta = \frac{m v^2}{r} \tag{1}$$

$$\Sigma F_y = N \cos\theta - f \sin\theta - m g = 0 \tag{2}$$

Let' s study these for a moment. The forces in the x direction are the horizontal components of the normal force and the frictional force. The frictional force acts down the plane and toward the center of the circle (as shown by the direction of the red arrow representing the centripetal force). This means that its horizontal component is directed toward the center of motion. Similarly, the horizontal component of the normal force is always in this direction. By convention, we call this the positive x direction (we could call it negative and get the same final results as long as we are consistent in our sign usage). Now, it is important to remember that the car is accelerating as it moves along the circular arc, and so these two forces combine to produce a centripetal force.

There is no acceleration in the y direction, so the sum of vertical forces is zero. These forces are the vertical component of the normal force (acting up), the weight (down) and the vertical component of friction (also down). Choosing signs carefully, we obtain equation (2) above.

Finally, we utilize the relationship $f = \mu N$ so that we can rewrite eqs. (1) and (2) as:

$$N (\sin\theta + \mu \cos\theta) = \frac{m v^2}{r} \tag{3}$$

$$N (\cos\theta - \mu \sin\theta) = m g \tag{4}$$

We could solve for N in eq. (e) and substitute that relationship into eq. (4), or we can divide the two equations (noting that the common factors of N and m will cancel), giving us :

$$\frac{(\sin\theta + \mu \cos\theta)}{\cos\theta - \mu \sin\theta} = \frac{v^2}{r g} \tag{5}$$

It is simple now to solve for v :

$$v = \sqrt{r g \frac{(\sin\theta + \mu \cos\theta)}{\cos\theta - \mu \sin\theta}} \tag{6}$$

Now, the second part of the problem asks what is the minimum speed the car can have and not slide down the bank. In this case, the force of friction acts up the plane (as shown by the orange arrow). The forces are the same as in the first part (normal force, weight, friction), but here the frictional force acts up the plane and away from the center of curvature. Thus, in this case, the vertical component of friction is positive, and the horizontal component of friction is negative. Newton' s second law applied here gives :

$$\Sigma F_x = N \sin\theta - f \cos\theta = \frac{m v^2}{r} \tag{7}$$

$$\Sigma F_y = N\cos\theta + f\sin\theta - m g = 0 \tag{8}$$

Setting f = μ N, we obtain :

$$N(\sin\theta - \mu\cos\theta) = \frac{m v^2}{r} \tag{9}$$

$$N(\cos\theta + \mu\sin\theta) = m g \tag{10}$$

Dividing equations (9) and (10) and solving for v, we find the minimum speed that will keep the car moving along the bank without sliding down :

$$v = \sqrt{r g \frac{(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}} \tag{11}$$

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2. The following is an actual exam question produced for the Chicago Public Schools by a nationally known educational consulting company : "Consider a package of weight W resting on a platform 1 meter above the surface of the Earth. What is the weight of the package if it is moved to a platform 3 m above the surface of the Earth?" The answer key provided by the nationally known (and highly, highly paid) company was W/9. (Really, I' m not making this up. Just walk up two meters and lose 89 % of your weight.) What "logic" did they employ to obtain this result? Now, as patiently as I did, please explain to them their error.

Solution : The framers of this question understood that weight is the force of the Earth' s gravity acting on an object. They also understood that the force of gravity varies as $1/R^2$. Their mistake was in using the surface of the Earth as the place where R = 0. So, making this error, proportional reasoning suggests that if you move three times farther away from the source of gravity, then the force of gravity acting on you will decrease by a factor of $1/3^2$ or 1/9. However, the source of the Earth’s gravity is the center of the Earth, some 6400 km distant; if an object starts 6400.001 km away from the center of the Earth and moves to a distance of 6400.003 km from the center of the Earth, its fractional change in distance is miniscule, and any change in weight will be so small as to be immeasurable.

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3. An astronaut orbits a newly discovered planet in the star system Ignatius 436 B. The astronaut discovers that the new planet has exactly the same radius as the Earth, but that the period of one revolution in low orbit is 44 minutes (the period of one revolution in low orbit around the Earth is 88 minutes). What is the density of the discovered planet? (The average density of the Earth is 5.5 g/cm^3 (or 5500 kg$/m^3$)).

Solution : Refer to your class notes for the derivation we did for Kepler' s third law. An object in orbit around a central mass will obey the relationship :

$$M P^2 = \left(4\pi^2/G\right) R^3$$

where M is the mass of the central object (not of the orbiting object), P is the period of one revolution, G is the Newtonian Gravitational constant, and R is the radius of the orbit. The most efficient way to solve this problem is using proportional reasoning. We can write this equation in terms of mass :

$$M = \left(\frac{4\pi^2}{G}\right)\frac{R^3}{P^2}$$

If the planets have the same radius, and we know G is a constant throughout the universe, then we can determine the mass of Ignatius in terms of the mass of the Earth. This equation shows that the mass of a planet is proportional to $1/P^2$. If the period of low orbit around Ignatius is 1/2 the period of orbit around the Earth, then the mass of Ignatius must be $1/(1/2)^2 = 4$ times the mass of the Earth.

Now, the question asks about density. Since we know that density = mass/volume, and we also know the volume of a sphere is dependent on its radius^3, it is easy to conclude that the two planets have the same volume. If Ignatius has four times the mass of the Earth and the same volume, then the density of Ignatius must be 4 times greater than the Earth, or a density of 22 g/cm^3.

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4. Multiple choice question 6, p. 221

Solution : If there were no air friction, all of the brick' s original potential energy would be converted to kinetic energy upon impact with the ground. However, if there is a force acting in the direction opposing motion, this force would do an amount of work equal to f H (where f is the frictional force and H is the height of the building) during the flight of the brick. Thus, some amount of the original potential energy went into work against friction, and the rest into kinetic energy. This means there is less kinetic energy at the end than potential at the beginning, and the correct choice is C.

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5. Conceptual question 6, p. 220

Solution : Let' s say we launch from the ground; in that case the initial potential energy is zero. (It doesn' t really matter where we launch, we can always say the potential energy is zero at our launch level. This is absolutely fine as long as we recognize the potential energy increases above the reference level, and decreases if we go below the reference level). If the initial velocity is v_o, we know the projectile has initial components of motion of $v_o \cos\theta$ and $v_o \sin\theta$ in the horizontal and vertical directions, respectively. Therefore, the initial speed of the object is :

$$v_0^2 = v^2 \cos_0^2\theta + v_0^2 \sin^2\theta$$

where I will write this in terms of the square of the speed. This is convenient since I don' t have to mess with square roots, and because what I really care about here is the kinetic energy which depends on the square of the speed. Now, we know that the conservation of energy tells us that the object' s energy will always be equal to this initial kinetic energy. As the object rises, some of the KE is converted to PE, and the higher the object rises, the more KE is converted to PE. However, if we are ignoring air friction, the horizontal component of velocity never changes, so the quantity $\frac{1}{2}$m $v_0^2 \cos^2\theta$ never varies during flight. It is only the vertical velocity that decreases, and thus the only decrease in KE comes from a reduction in the decrease of the vertical velocity.

At the apex of motion, the vertical velocity is zero but the horizontal velocity is still equal to its original value. Therefore, any increase in potential energy can arise only from a reduction of vertical velocity; thus, the greater the initial vertical velocity, the higher the object can rise. If a series of objects are launched at different angles with the same launch speed, the object with the greatest initial vertical velocity (in other words, the one shot at the greatest angle to the Earth) can rise the highest.

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6. Multiple choice question 10, p. 221. For each option, state whether the statement is true or false, and then provide an explanation of your reasoning (saying only T or F will yield no credit.)

Solution : Option A is false. Potential energy = m g h. If both objects have 100 J of PE and different masses, the lighter object must be at a greater height in order that the product of m g h be the same for the two masses.

Option B : Both objects start and end with 100 J. The conservation of energy requires that this statement be true.

Option C : If we neglect air friction, the total energy is the same throughout the flight. If they are released from rest, they have zero KE at the start. Therefore, each mass will convert 100 J to final KE. Since we know that

$$KE_{final} = \frac{1}{2} m v^2 \Rightarrow v_{final} = \sqrt{2 \frac{KE_f}{m}}$$

The final KE for both is 100 J, so we can see that the final speed must be different since the masses are different. Option C is false. We could have also used the result from option A; if the objects started at different heights, they will have different speeds upon impact.

Option D is also true if we neglect friction. The force acting on each object is its weight, and this produces an acceleration. Newton' s second law tells us :

$$\Sigma F = m g = m a \Rightarrow a = g$$

and this results holds independent of mass.

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7. Problem 8, p. 222

Solution : The work done by each force is the product of the magnitude of the component of force acting in the direction of displacement times the displacement, or

$$W = F \cos\theta \cdot s$$

The normal force does no work, since that force acts perpendicular (normal) to the plane of motion. The component of gravity down the plane is m g sin θ, so the work done by gravity is :

$$W_{grav} = m g \sin\theta \cdot s = 8\,kg \cdot 9.8\,m/s^2 \cdot \sin 53 \cdot 2\,m = 125\,J$$

The force of friction acts up the plane with magnitude μ m g cos θ, so the work done by friction is :

$$W_f = -\mu m g \cos\theta\, s = -38\,J$$

The total work done is $125\,J - 38\,J = 87\,J$

We could have obtained this result by directly computing the work done by the total force acting along the plane :

$$W_{total} = (m\,g \sin\theta - \mu\,m\,g \cos\theta) \times s$$

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8. A string of radius 5 m is attached to an object of 10 kg. The object moves in a horizontal circle completing 1 revolution every 2 s. How much work is done on the object by the string? Explain your answer fully.

Solution : The string does no work on the mass. If the object is moving in a horizontal circle, its velocity vector is always perpendicular to the radius of its orbit. Since tension acts along the radius, the tension is perpendicular to the motion of the mass at all times. Since work is the product of the force in the direction of displacement times the displacement, the work done is zero, since there is no force in the direction of displacement.

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9. Problem 18, p. 223.

Solution : We use the work - energy theorem to approach these questions :

The 1.5 kg book slows from 3.21 m/s to 1.25 m/s. Its KE has decreased, so we know negative work (probably by friction) has been done on the book. We find the work done from the change in KE :

$$W = \Delta KE = \frac{1}{2}\,m\,v_f^2 - \frac{1}{2}\,m\,v_o^2 = \frac{1}{2}\,m\left(v_f^2 - v_o^2\right) = \frac{1.5\,kg}{2}\left((1.25\,m/s)^2 - (3.21\,m/s)^2\right) = -6.56\,J$$

In the next part, we are told negative work is done on the block, and we are asked to find the speed of the block after the negative work has been done. Again, the work energy theorem tells us :

$$W = \Delta KE$$

Here, W = - 0.75 J, so :

$$-0.75\,J = 1.5\,kg/2\left(v_f^2 - (1.25\,m/s)^2\right)$$

$$v_f^2 = -\frac{2 * 0.75\,J}{1.5\,kg} + (1.25\,m/s)^2 \Rightarrow v_f = 0.75\,m/s$$

If the work done at point B were + 0.75 J, then the final speed would be :

$$v_f^2 = +\frac{2 * 0.75\,J}{1.5\,kg} + (1.25\,m/s)^2 \Rightarrow v_f = 1.60\,m/s$$

The diagram in problem 1 was drawn using the Mathematica program that is on every Loyola network computer. If you are interested in seeing how such a program is written, I include the code below. I could have used fewer statements by combining several ' Graphics' statements, but I prefer to write them out explicitly. Here is what it takes to produce the graphic in problem 1, I will be happy to show anyone how to get started with Mathematica and how this code works :

In[665]:=
```
g1 = Graphics[Line[{{0, 0}, {20, 0}}]];
g2 = Graphics[Line[{{0, 0}, {15, 15}}]];
g3 = Graphics[
   {Opacity[0.2], Blue, Rotate[Rectangle[{8, 8}, {12, 12}], 45 Degree, {Left, Bottom}]}];
g4 = Graphics[{Blue, Thickness[0.01], Arrow[{{8, 10.8}, {8, 2.8}}]}];
g5 = Graphics[{Cyan, Thickness[0.01], Arrow[{{8, 10.8}, {1.8, 17}}]}];
g6 = Graphics[{Orange, Thickness[0.01], Arrow[{{2.4, 5.2}, {6.6, 9.4}}]}];
g7 = Graphics[Text[StyleForm["Weight", FontSize → 14, FontWeight → "Bold"],
     {9, 6.8}, {0, 1}, {0, -1}]];
g8 = Graphics[Text[StyleForm["Normal", FontSize → 14, FontWeight → "Bold"],
     {6, 13}, {0, -1}, {1, -1}]];
g9 = Graphics[Text[StyleForm["Friction", FontSize → 14, FontWeight → "Bold"],
     {4.5, 7.2}, {0, 1}, {1, 1}]];
g10 = Graphics[{Red, Thickness[0.01], Arrow[{{8, 10.8}, {1, 10.8}}]}];
g11 =
  Graphics[Text[StyleForm["m v^2/r", FontSize → 14, FontWeight → "Bold"], {3.5, 11.5}]];
g12 = Graphics[Text[StyleForm["Friction", FontSize → 14, FontWeight → "Bold"],
     {11.2, 14.2}, {0, 1}, {1, 1}]];
g13 = Graphics[{Green, Thickness[0.01], Arrow[{{13.5, 16.3}, {9.3, 12.1}}]}];
g14 = Graphics[Circle[{0, 0}, 6, {0, π / 4}]];
g15 = Graphics[Text[StyleForm["θ", FontSize → 24, FontWeight → "Bold"], {6.5, 2.5}]];
g16 = Graphics[{Dashed, Line[{{8, 10.8}, {8, 18.8}}]}];
g17 = Graphics[Circle[{8, 10.8}, 6, {3 π / 4, π / 2}]];
g18 = Graphics[Text[StyleForm["θ", FontSize → 24, FontWeight → "Bold"], {5, 17.5}]];
Show[g1, g2, g3, g4, g5, g6, g7, g8, g9, g10, g11, g12, g13, g14, g15, g16, g17, g18]
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