1. This is a conservation of energy problem. We equate all the energy at the beginning to all the energy at the end; this will allow us to determine an expression for the velocity of the masses just as the larger mass hits the ground. At the beginning (defined as the moment just before motion ensues), there is no kinetic energy, and the gravitational potential derives from the height of the larger mass. At the end, the larger mass has zero PE and the smaller mass has risen a height \( d \). At the end, both masses are moving at the same velocity \( v \), so we have (subscript “i” refers to initial values; subscript “f” refers to final values):

\[
\begin{align*}
KE_i &= 0 \\
U_i &= m_2 gd \\
KE_f &= \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 \\
U_f &= m_1 gd
\end{align*}
\]

Equating energy at the beginning to energy at the end:

\[
m_2 gd = m_1 gd + \frac{1}{2} (m_1 + m_2) v^2
\]

Solving for \( v \) yields:

\[
v = \sqrt{\frac{2 (m_2 - m_1) gd}{m_1 + m_2}}
\]

2. This is a conservation of energy problem with no friction. Because there are no dissipative forces, we know the system always has the total initial energy, which is \( \frac{1}{2} k_1 x_1^2 \) where \( k_1 \) is the spring constant of the spring on the left and \( x_1 \) is the distance it is compressed. When the box completely compresses the spring on the right, the box has no kinetic energy, and the total energy of the system is the elastic potential of spring \( k_2 \), namely \( \frac{1}{2} k_2 x_2^2 \). Conservation of energy allows us to equate the initial and final energies:

\[
\frac{1}{2} k_1 x_1^2 = \frac{1}{2} k_2 x_2^2
\]

Solving for the compression on the right, \( x_2 \):

\[
x_2 = x_1 \sqrt{\frac{k_1}{k_2}} = 0.04 \text{ m} \sqrt{\frac{32 \text{ N/m}}{16 \text{ N/m}}} = 0.056 \text{ m}
\]

The maximum speed of the box will occur once the spring has reached its equilibrium position. For this part of the problem, the initial state corresponds to the left spring completely compressed, and
the final state occurs when the box is moving at speed \( v \) and the spring has reached its equilibrium position. Equating initial and final energies:

\[
\frac{1}{2} k_1 x_1^2 = \frac{1}{2} m v^2 \Rightarrow v = x_1 \sqrt{k_1 / m} = 0.04 \text{ m} \sqrt{32 \text{ N/m} / 1.5 \text{ kg}} = 0.18 \text{ m/s}
\]

3. For this problem, it is helpful to refer to the diagram:

![Diagram](image)

**Vibrating simple pendulum**

where mass at \( E_2 \) makes an angle of \( \theta \) (here \( \theta = 42^\circ \)) with respect to the vertical. If the length of the string is \( L \), the vertical height of \( E_2 \) above the lowest position (\( M \) in this diagram) is \( L - L \cos \theta \) or \( L(1 - \cos \theta) \). Thus, the potential energy of the child is:

\[
\text{PE} = m g h = m g L (1 - \cos \theta) = 25 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 2.2 \text{ m}(1 - \cos 42) = 138 \text{ J}
\]

Initially, the total energy in the swing is 138 J. When the swing reaches the bottom of its arc, the PE is zero and now all the initial PE has converted to KE, so we have:

\[
\frac{1}{2} m v_{\text{bottom}}^2 = 138 \text{ J} \Rightarrow v_{\text{bottom}} = \sqrt{2 \cdot 138 \text{ J} / 25 \text{ kg}} = 3.3 \text{ m/s}
\]

4. This is a simple conservation of energy problem. The potential energy of the hailstone in the cloud is converted to kinetic energy just before impact. This yields:

\[
m g h = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{2 g h} = \sqrt{2 \cdot 9.8 \text{ m/s}^2 \cdot 500 \text{ m}} = 99 \text{ m/s}
\]

This is equivalent to a speed of over 200 mi/hr. So clearly the answer to the second question is no; raindrops/hailstones don’t hit the ground at Mach 0.67. There is significant air friction that acts on the falling raindrops. (Falling raindrops are not spherical in shape; rather, air friction causes them to take on the shape of a hamburger bun.)
5. This is a case where there is work done by friction. We know (eq. 7 - 16) that the total final energy equals the total initial energy plus any work done by other (non-conservative) forces, or:

\[ E_f = E_i + W_{\text{other}} \]

Here, the final energy is zero, since the block has stopped moving and the spring has relaxed to its equilibrium position. At the beginning, the total energy is potential, so we have:

\[ 0 = \frac{1}{2} k x^2 + W_{\text{other}} \]

This allows us to quickly determine that the work due to friction is:

\[ W_{\text{friction}} = -\frac{1}{2} k x^2 = -\frac{1}{2} \cdot 100 \text{ N/m} \cdot (0.2 \text{ m})^2 = -2 \text{ J} \]

We also know that \( W = F s \), and in this case, the force is due to friction so that \( F = \mu N = \mu mg \).

Thus we have:

\[ \mu mg s = W_{\text{other}} \Rightarrow \mu = \frac{W_{\text{other}}}{mg s} = \frac{2 \text{ J}}{0.5 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 1 \text{ m}} = 0.4 \]

6. This is a problem that combines conservation of energy with kinematics. If the hill is frictionless, we know that the total mechanical energy at the bottom will equal the total mechanical energy at the top. Some of you spoke with me indicating you assumed that the sled would reach zero velocity at the top. This is not an assumption you can make without calculating the KE at the bottom compared to the PE at the top. At the bottom, the sled has a KE given by

\[ KE_{\text{bottom}} = \frac{1}{2} m v_{\text{bottom}}^2 \]

If this is greater than the PE at the top (\( mg H \)), the sled will not have converted all its initial KE to PE and thus will have some remaining KE. We can use energy conservation to show:

\[ \frac{1}{2} m v_{\text{bottom}}^2 = mg H + \frac{1}{2} m v_{\text{top}}^2 \]

or

\[ v_{\text{top}} = \sqrt{v_{\text{bottom}}^2 - 2gH} \]

So, if:

\[ v_{\text{bottom}}^2 > 2gH \]

the sled will still be moving at the top. For our parameters, we have:

\[ v_{\text{bottom}}^2 = (22.5 \text{ m/s})^2 = 506 \text{ (m/s)^2} \]

\[ 2gH = 2 \cdot 9.8 \text{ m/s}^2 \cdot 11 \text{ m} = 216 \text{ (m/s)^2} \]

so the speed of the sled at the top is

\[ v_{\text{top}} = \sqrt{506 - 216} \text{ (m/s)^2} = 17 \text{ m/s} \]
Now, if an object falls from a height $H$, it will take a time (see earlier kinematics problems)

$$t = \sqrt{\frac{2H}{g}}$$

to hit the ground. In the absence of air friction, the object will have constant horizontal velocity, so the total distance traveled before impact will be simply:

$$\text{distance traveled} = v_{\text{top}} \sqrt{\frac{2H}{g}} = 17 \text{ m/s} \sqrt{2 \cdot 11 \text{ m/} 9.8 \text{ m/s}^2} = 25.4 \text{ m}$$

7. This problem combines conservation of energy with Newton's second law applied to rotational motion. In the absence of friction, the total energy when the car starts must equal the total energy when the car is at the top of the loop. Using subscripts "i" to refer to initial parameters and "t" to refer to the values at the top of the loop, we have:

$$\text{KE}_i = 0 \quad \text{U}_i = m \cdot g \cdot H$$

$$\text{KE}_t = \frac{1}{2} m \cdot v_t^2 \quad \text{U}_t = 2 \cdot m \cdot g \cdot R \quad \text{(since you are 2 R above the ground)}$$

Now, a common assumption is that the speed at the top is zero; however, we showed in class that the car must have enough speed to stay in contact with the track. If we apply Newton's second law to the top of the loop, we have the normal force acting down, the weight acting down, and these must combine to produce a centripetal force acting (down) toward the center of the circle. In this case, let's call down the positive direction, then:

$$N + m \cdot g = m \cdot v_t^2 / R$$

The car leaves the track when $N = 0$, so we can find the minimum speed the car must have at the top to complete the loop by setting $N = 0$ and obtaining:

$$v_t^2 = R \cdot g$$

Now, using this result in the conservation of energy gives us:

$$m \cdot g \cdot H = \frac{1}{2} m \cdot v_t^2 + 2 \cdot m \cdot g \cdot R = \frac{1}{2} m \cdot g \cdot R + 2 \cdot m \cdot g \cdot R$$

Cancelling out common factors of $m$ and $g$, we get the result:

$$H = \frac{5}{2} \cdot R$$

This is a well known result; a sliding object must start at a height equal to 2.5 times the radius of the loop to complete the loop.