PHYS 111
HOMEWORK #11

Due : 1 Dec. 2016

1. p. 250 no. 22

Solution : This problem combines conservation of momentum and conservation of energy. This problem is similar in scope to the discussion of the ballistic pendulum.

If our system consists of the two blocks, there are no external forces acting on the blocks along the level surface, so we can apply the conservation of momentum to their interaction. This tells us that the total momentum before collision is equal to the total momentum after collision or :

$$M V_i = (M + M) V_f \Rightarrow V_f = \frac{1}{2} V_o$$

Since the initial velocity of the first block was 5m/s, the speed of the combined mass will be 2.5 m/s. Now, the combined mass of 4kg moves up a frictionless plane. Here, there is an external force acting on the mass (gravity), but since gravity is a conservative force, we can use the conservation of energy. We know that the kinetic energy at the bottom of the ramp will equal the potential energy at its highest point, or:

$$\frac{1}{2}(2\,M)\,V^2 = (2\,M)\,g\,h \Rightarrow h = \frac{V^2}{2\,g}$$

where V is the speed of the combined mass at the bottom of the ramp (which we just found to be 2.5m/s). Using this value we find that:

$$h = \frac{(2.5\,m/s)^2}{2\left(9.8\,m/s^2\right)} = 0.32\,m$$

Finally, the problem asks us to find how far the mass travels along the incline. We just determined the height of the mass above the ground at its highest point, and this height is related to distance along the incline by :

$$\sin\theta = \frac{h}{L} \Rightarrow L = \frac{h}{\sin\theta} = \frac{0.32\,m}{0.71} = 0.45\,m$$

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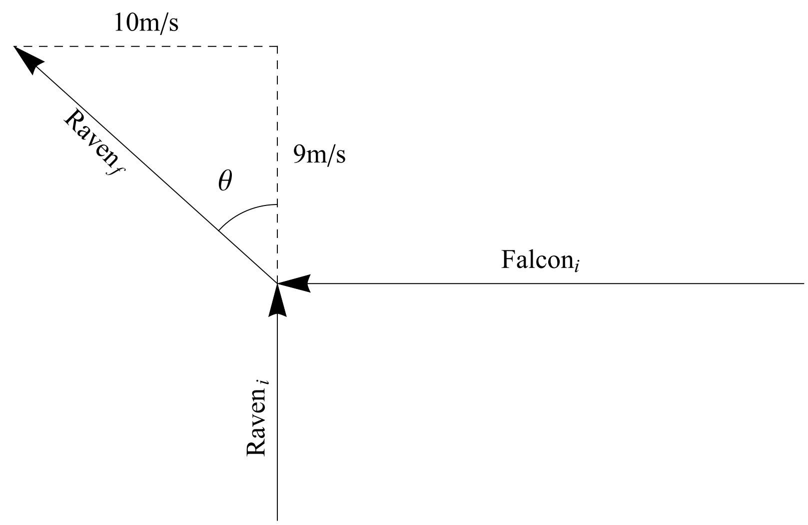
2. p. 250, no. 26

Solution : Let' s consider the diagram below to guide our work on this problem.



(I have omitted the vector for the falcon's motion after collision.)

We are asked to find the angle through which the raven is deflected. Our goal will be to find the x and y components of the raven's velocity after collision. We will do this by employing conservation of momentum.

Here we have conservation of momentum in two directions. We are told the falcon and raven are traveling at right angles to each other. Let' s just arbitrarily say the falcon is moving in the + x direction and the raven is moving in the + y direction. Conservation of momentum tells us that the momentum is conserved in each dimension. (In the equations below, the subscript i refers to initial conditions, and the subscript f will refer to final conditions.) Initially (before the collision), the falcon has momentum in the + x direction given by :

$$p_{x,i} = m_{falcon}\, v_{falcon,\,i} = 0.6\,kg \cdot 20\,m/s = 12\,kg\,m/s$$

and in the y direction, the raven has initial momentum of:

$$p_{y,i} = m_{raven}\, v_{raven,\,i} = 1.5\,kg \cdot 9\,m/s = 13.5\,kg\,m/s$$

After collision, the falcon has motion in the -x direction, and the raven has motion in both the x and y directions. To conserve momentum in each direction, we write:

$$p_{x,i} = p_{x,f} \Rightarrow 12\,kg\,m/s = m_{falcon}\, v_{falcon,\,f} + m_{raven}\, v_{raven,\,x,\,f}$$

We are given that the final velocity of the falcon is 5m/s in the -x direction, so the x momentum equation becomes:

$$12\,kg\,m/s = (0.6\,kg)\,(-5\,m/s) + 1.5\,kg \cdot v_{raven,\,x,\,f}$$

Simple algebra yields that the raven has an x component of final velocity of 10m/s.

Now, in the y direction :

$$p_{y,i} = p_{y,f} \Rightarrow 13.5\ \text{kg m/s} = m_{raven}\, v_{raven,\,y,\,f}$$

or

$$13.5\ \text{kg m/s} = 1.5\ \text{kg}\ v_{raven,\,y,\,f} \Rightarrow v_{raven,\,y,\,f} = 9\ \text{m/s}$$

These calculations shows that the raven's final velocity has an x component of 10 m/s and a y component of 9m/s. Using the diagram above, we can write:

$$\tan\theta = \frac{10\ \text{m/s}}{9\ \text{m/s}} \Rightarrow \theta = \tan^{-1}\left(\frac{10}{9}\right) = 48^{o}$$

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3. p. 250, no. 30

Solution : This is the simplest form of elastic collision : a head on collision with one mass initially stationary. This situation is describe by equations 8.11 and 8.12 as derived in class :

In this case, the 2.5g cent (it is properly called a cent, not a penny) is mass a and the 5 g nickel is the initially stationary mass b. Thus we have after collision:

$$v_{A,f} = \frac{m_A - m_b}{m_A + m_B} v_{A,i} = \frac{2.5\ \text{g} - 5\ \text{g}}{2.5\ \text{g} + 5\ \text{g}}(2.2\ \text{m/s}) = -0.73\ \text{m/s}$$

$$v_{B,f} = \frac{2\, m_A}{m_a + m_B} v_{A,i} = \frac{2\,(2.5\ \text{g})}{7.5\ \text{g}}(2.2\ \text{m/s}) = 1.47\ \text{m/s}$$

The smaller mass rebounds at 0.73 m/s, and the larger mass moves in the original direction of A at 1.47 m/s. We can verify that kinetic energy is conserved :

$$KE_{before\,=} \frac{1}{2} m_A\, v^2{}_{A,i} = \frac{1}{2}(2.5\ \text{g})\,(2.2\ \text{m/s})^2 = 6.1\ \text{g m}^2/\text{s}^2$$

$$KE_{after\,=} \frac{1}{2} m_A\, v_{A,f}^2 + \frac{1}{2} m_B\, v_{B,f}^2 =$$

$$\frac{1}{2}\left(2.5\ \text{g}\,(0.73\ \text{m/s})^2\right) + \frac{1}{2}\left(5\ \text{g}\,(1.47\ \text{m/s})^2\right) = 0.66\ \text{g m}^2/\text{s}^2 + 5.40\ \text{g m}^2/\text{s}^2 = 6.1\ \text{g cm}^2/\text{s}^2$$

and the final kinetic energy equals the original kinetic energy.

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4. p. 250, no. 32

Solution : This is an example of an elastic collision, so both momentum and energy are conserved. It is instructive to start by pointing out the most frequently offered incorrect answer to part a) (what is the maximum energy that can be stored in the springs?). The most common incorrect answer computes the initial kinetic energy of the system before collision (in this case that is 4 J), and then asserts that is the maximum amount of energy that can be stored in the springs. Why is this incorrect? This answer violates conservation of momentum. If all the energy is transferred to the springs, then for at least that moment there is no kinetic energy in the masses; that means that both masses are simultaneously stationary which implies the momentum is zero.

In a collision of this form (head-on, elastic collision, one object initially stationary), we know that the final velocities are related to the initial velocity according to:

$$v_{A,f} = \frac{m_A - m_b}{m_A + m_B} v_{A,i} \text{ and } v_{B,f} = \frac{2\, m_A}{m_a + m_B} v_{A,i} \quad (1)$$

In this problem, object A (the object initially moving) is less massive than the stationary target B. This means that the final velocity of A will be negative (since m_A - m_B will be negative). This tells us that there is a moment when object A must come to rest in order to change direction of motion. Since momentum is always conserved, we can calculate the speed of mass B at this moment:

$$m_B V_B \text{(during collision)} = 2\text{ kg} \cdot 2\, m/s \Rightarrow v_B \text{(during collision)} = 0.4\, m/s$$

Knowing the speed of B at the moment A is stationary allows us to compare the KE in B at that moment with the initial KE of A:

$$\text{Initial KE of A} = \frac{1}{2} m_A v_{A,i}^2 = \frac{1}{2}(2\text{ kg})(2\text{ m/s})^2 = 4\text{ J}$$

$$\text{KE of B during collision} = \frac{1}{2} m_B v_B^2 = \frac{1}{2}(10\text{ kg})(0.4\text{ m/s})^2 = 0.8\text{ J}$$

We can see from these calculations that in order to conserve both energy and momentum during the collision, the amount of energy stored in the springs must be 3.2J.

Part b) asks us to use equations (1) to solve for the final velocities:

$$V_{A,f} = \frac{2\text{ kg} - 10\text{ kg}}{2\text{ kg} + 10\text{ kg}} \cdot 2\text{ m/s} = -1.33\text{ m/s}$$

$$B_{B,f} = \frac{2\,(2\text{ kg})}{2\text{ kg} + 10\text{ kg}} \cdot 2\text{ m/s}\; 0.667\text{ m/s}$$

Checking that KE is conserved:

$$KE_F = \frac{1}{2}\left(m_A v_{A,f}^2 + m_B v_{B,f}^2\right) = \frac{1}{2}\left(2\text{ kg} \cdot (-1.33\text{ m/s})^2 + 10\text{ kg}\,(0.667\text{ m/s})^2\right) = 4\text{ J}$$

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5. p. 251, no. 34

Solution : This problem involves the impulse - momentum theorem. We recall that impulse is defined as the product of force and the time interval in which it acts :

$$\text{Impulse} = F\,\Delta t$$

and this is equal to the change in momentum:

$$F\,\Delta t = \Delta(m\,v)$$

In this case, a 0.145 kg object changes its speed from 36m/s to 0m/s in 0.020s. The impulse equals the change in momentum:

$$\Delta(m\,v) = 0.145\text{ kg}\,(36\text{ m/s}) = 5.22\text{ kg m/s}$$

If the force is exerted over a period of 20 ms (0.020s), the force acting on the catcher is

$$F = \frac{\Delta(m\,v)}{\Delta t} = \frac{5.22\text{ kg m/s}}{0.02\text{ s}} = 261\text{ N}$$

The question asks for the force exerted on the catcher, so the direction of the force is in the direction of motion of the baseball.

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6. p. 251, no. 40

Solution : The impulse equals the change in momentum of the bullet which is :

$$\Delta(m\,v) = 0.0085\text{ kg}\cdot 900\text{ m/s} = 7.65\text{ kg m/s}$$

If the force is exerted over a time interval of 0.002s, the force is:

$$F = \frac{\Delta(m\,v)}{\Delta t} = \frac{7.65\text{ kg m/s}}{0.002\text{ s}} = 3825\text{ N} = 860\text{ lbs (using 1 N} = 0.22\text{ lbs)}$$

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7. p. 253, no. 70

Solution : In this case, we have two identical carts moving toward each other at the same speed. This means that the total momentum of the system is zero. Since momentum is conserved in all collisions, the total momentum of the system is always zero.

We are presented with two conditions : one in which the velcro ends of the carts collide, and one in which the springs collide. The velcro case is clearly inelastic and the two carts will stick together to form one mass. Since the total momentum is zero, the velocity of this combined mass must be zero.

The wording of the problem implies without stating explicitly that the spring collision is elastic. This means the two carts must conserve both momentum and energy after collision. Since the momentum is zero, the two carts must move apart from each other at the same speed; in other words, each car approaches with a speed v, and then rebounds at the same speed v.

(Two questions for further consideration: 1) If we use equations (1) above, we do not reproduce these results, why not? 2) Try to write out the equations of momentum and energy conservation for the elastic case and reproduce the results we obtained by our reasoning.)

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8. p. 277, no. 10.

Solution : We are asked to find the angular speeds of two fans, one which started from rest and one which started with an initial angular velocity. We use the relationship between angular velocity and angular acceleration :

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t} \Rightarrow \omega_f = \omega_i + \alpha\, t$$

These equations are analogous to the linear versions: ($v_f = v_i$ + a t).

For the fan that is slowing down, we can write:

$$\omega_1(t) = \omega_o + \alpha\, t = 200\, \text{rad}/\text{s} - 20\, \text{rad}\big/\text{s}^2\, t$$

(the minus sign arises because the rotational rate is slowing down). For the fan starting from rest, we have :

$$\omega_2(t) = \omega_o + \alpha\, t = 0 + 60\, \text{rad}\big/\text{s}^2\, t$$

To find the time when the angular speeds are equal, set $\omega_1 = \omega_2$:

$$\omega_1(t) = \omega_2(t) \Rightarrow 200\, \text{rad}/\text{s} - 20\, \text{rad}\big/\text{s}^2\, t = 60\, \text{rad}\big/\text{s}^2\, t$$

$$t = \frac{200\, \text{rad}/\text{s}}{80\, \text{rad}\big/\text{s}^2} = 2.5\, \text{s}$$

Use t = 2.5 s in the angular velocity equations to find that both fans have a speed of 150rad/s at t = 2.5 s:

$$\omega_1(2.5\, \text{s}) = 200\, \text{rad}/\text{s} - 20\, \text{rad}\big/\text{s}^2\, (2.5\, \text{s}) = 150\, \text{rad}/\text{s}$$

$$\omega_2(2.5\, \text{s}) = 60\, \text{rad}\big/\text{s}^2\, (2.5\, \text{s}) = 150\, \text{rad}/\text{s}$$

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9. p. 277, no. 12

Solution : We are asked to find the angular acceleration of a CD :

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{1600\, \text{rpm} - 570\, \text{rpm}}{133\, \text{m}} = 7.74\, \text{rev}\big/\text{min}^2$$

$$= 7.74\, \text{rev}\big/\text{min}^2\, (2\,\pi\, \text{rad}/\text{rev}) \cdot \left(\frac{1\, \text{min}}{60\, \text{s}}\right)^2 = 0.013\, \text{rad}\big/\text{s}^2$$

Alternately, you could have converted (1600 rpm - 570 rpm) to 108 rad/s and 133 min to 7980 s to compute the same angular acceleration.

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10. p. 277, no. 18.

Solution : A wheel has an initial angular velocity of 12 rad/s and comes to rest after moving through an angular displacement of 60 rad. We are asked to find the angular acceleration. We use :

$$\omega_f^2 = \omega_o^2 + 2\,\alpha\,\theta$$

(This is analogous to $v_f^2 = v_o^2 + 2\, a\, s$).

For our values:

$$0 = (12\,\text{rad}/\text{s})^2 + 2\,\alpha\,(60\,\text{rad}) \Rightarrow \alpha = -\frac{144\,\text{rad}^2/\text{s}^2}{2\cdot 60\,\text{rad}/\text{s}} = -1.2\,\text{rad}/\text{s}^2$$

We use the definition of angular acceleration to compute the time:

$$\alpha = \frac{\omega_f - \omega_o}{t} \Rightarrow t = \frac{\omega_f - \omega_o}{\alpha} = \frac{0 - 12\,\text{rad}/\text{s}}{-1.2\,\text{rad}/\text{s}^2} = 10\,\text{s}$$