PHYS 11
HOMEWORK #12

Due : 6 Dec. 2016

1. Problem 24, p. 277

Solution : We are asked to find the radial, tangential and total acceleration for a point on the edge of a flywheel at different times. We are told that the angular acceleration is constant, therefore we know that all points on the edge of the wheel will have a tangential component of acceleration given by :

$$a_{\tan} = r\,\alpha = (0.300\,\text{m})\left(0.600\,\text{rad}/\text{s}^2\right) = 0.180\,\text{m}/\text{s}^2$$

where r is the radius of the wheel and α the angular acceleration This will be the tangential acceleration of a point on the edge of the wheel at all times.

The radial acceleration is given by:

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2\,r \;\;(\text{since } v = r\,\omega)$$

where ω is the instantaneous angular velocity. Since we are asked to find ω as a function of angular displacement, the most convenient equation for us is :

$$\omega_f^2 = \omega_o^2 + 2\,\alpha\,\theta$$

In our case, we are told the wheel starts from rest, so ω_o is zero, so we have:

$$\omega_f^2 = 2\,\alpha\,\theta$$

and the radial acceleration for any value of θ can be written as:

$$a_{\text{rad}} = \omega^2\,r = 2\,\alpha\,\theta\,r$$

Recall that α and r are fixed ($\alpha = 0.600$ rad$/s^2$ and r = 0.300m). We can easily compute the radial acceleration for points on the wheel when $\theta = 0$, 60^o ($=\pi/3$ rad) and 120^o ($= 2\pi/3$ rad) (make sure you use the radian value for θ):

$$a_{\text{rad}}\;(\theta = 0) = 0$$

$$a_{\text{rad}}\;(\theta = \pi/3) = 0.377\,\text{m}/\text{s}^2$$

$$a_{\text{rad}}\;(\theta = 2\,\pi/3) = 0.754\,\text{m}/\text{s}^2$$

We find the magnitude of total acceleration by substituting the appropriate values into:

$$a_{\text{Total}} = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2}$$

2. Problem 28, p. 278

Solution : The rotational KE of a rigid body is :

$$K = \frac{1}{2} I \omega^2$$

where I is the moment of intertia and ω the angular velocity. Since the angular velocities are the same, the ratio of kinetic energies will be equal to the ratio of moments of inertia. As we showed in class, the moment of inertia of a rod about its center is 1/12 ML^2; the moment of inertia of a rod about an end is 1/3 ML^2, so the ratio of kinetic energy is (1/12)/(1/3) = 1/4.

3. Problem 31, p. 278 (all parts, each part worth 5 points).

Solution : The moment of inertia for each case will be :

$$I = \sum_{i=1}^{4} m_i r_i^2$$

where m_i is the mass of each sphere and r_i is the distance of each sphere from the rotation axis. In the diagram below, the points represent the masses (each of 0.200 kg); the solid lines are the massless connecting rods, and the dashed lines show us how to compute the distance from the rotation axis to each mass. (The rotation axis is perpendicular to the page, going through the point in the center of the square.)

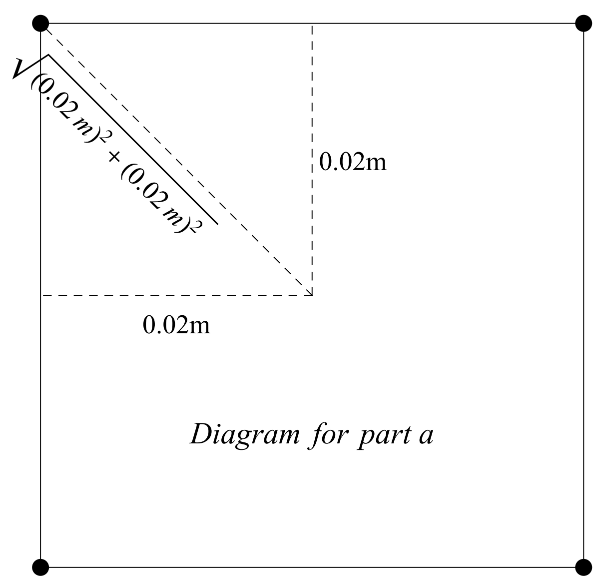

Diagram for part a

For this position of the rotation axis, the distance between each sphere and the rotation axis is given by the Pythagorean theorem:

$$r^2 = (0.200\text{ m})^2 + (0.200\text{ m})^2 = 0.080\text{ m}^2$$

Then, the moment of inertia for part a) is :

$$I = 4\,(0.200\text{ kg})\left(0.080\text{ m}^2\right) = 0.064\text{ kg m}^2$$

In part b), the rotation axis is along the line cutting through the middle of the square:

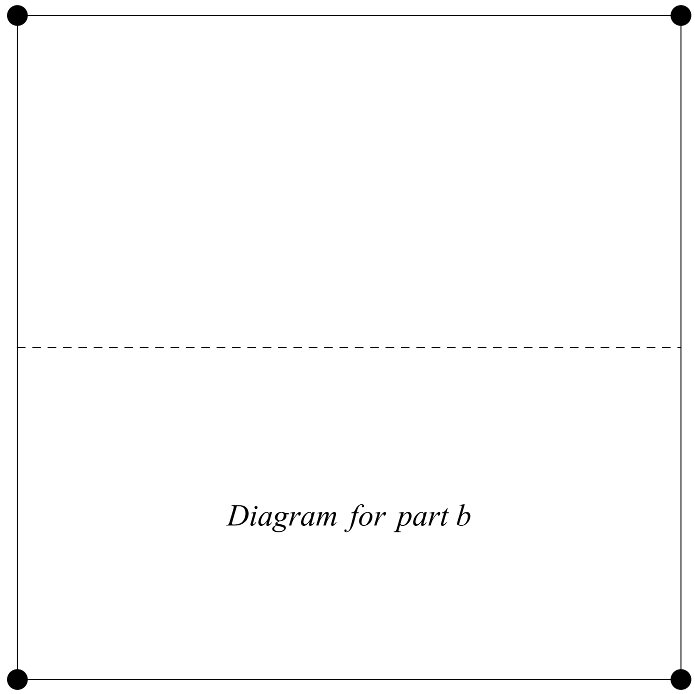

Diagram for part b

And each mass is 0.2m from the rotation axis, so we have:

$$I = 4\,(0.200\text{ kg})\,(0.200\text{ m})^2 = 0.032\text{ kg m}^2$$

In part c), the diagonal axis is the rotational axis. The two masses lying along the rotational axis contribute nothing to the moment of inertia; the other two masses are a distance $\sqrt{(0.2\,m)^2 + (0.2\,m)^2}$ away from the axis. The moment of interia in this case is:

$$I = 2\,(0.200\text{ kg})\left(0.08\text{ m}^2\right) = 0.032\text{ kg m}^2$$

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4. Problem 40, p. 278

Solution : We approach this using the conservation of energy, remembering that there will be both translational and rotational energy once the mass is falling. Before the mass falls, the mass possess gravitational potential energy (GPE); once it has fallen through a distance h, it converts that GPE to the translational kinetic energy of the falling mass, and to rotational kinetic energy of the spinning wheel. We can describe this as :

$$m\,g\,h = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2$$

We are told the wheel is a solid cylinder, so we know its moment of inertia is 1/2 M R^2 (where M is the mass of the cylinder, R is its radius, and m will be the mass of the descending object.) Since the string doesn't stretch or compress, the tangential velocity of string on the edge of the wheel must equal the speed at which the mass descends, which allows us to write:

$$v = R\omega$$

Substituting these into our energy equation gives:

$$m g h = \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{1}{2} M R^2 \left(\frac{v}{R}\right)^2\right)$$

$$\Rightarrow m g h = \frac{1}{2} v^2 \left(m + \frac{M}{2}\right)$$

We are asked to use the data given to solve for the mass of the cylinder, M :

$$M = 2\left(\frac{2 m g h}{v^2} - m\right)$$

for the provided values of m = 3 kg, h = 2.5m, v = 3.5 m/s, we obtain that M = 18 kg.

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5. Problem 46, p. 279

Solution : This is another conservation of energy problem. At the base of the hill, each object has kinetic energy; as each object rises up the hill, its initial KE is converted to GPE. If each object has a linear speed v at the base, the total KE can be written as :

$$K = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

Since the objects are rolling without slipping, we know that v is related to ω through $v = R\,\omega$ (where R is the radius of each object). Writing the moment of inertia as $\beta M R^2$, we can express the kinetic energy as:

$$K = \frac{1}{2} M v^2 + \frac{1}{2}\left(\beta M R^2 \left(\frac{v}{R}\right)^2\right) = \frac{1}{2} M v^2 (1 + \beta)$$

This total initial kinetic energy is converted to GPE, so that:

$$\frac{1}{2} M v^2 (1 + \beta) = M g h \Rightarrow h = \frac{v^2 (1 + \beta)}{2 g}$$

(Note that if $\beta = 0$ (there is no rotational energy), we get the same answer we did for objects sliding up a plane with no friction.) For a sphere, the moment of inertia is 2/5 M R^2 (so $\beta = 2/5$), and for a spherical shell I = 2/3 M R^2 ($\beta = 2/3$). Substitute these values for β to find that:

$$h_{sphere} = \frac{7}{10}\frac{v^2}{g} \qquad h_{shell} = \frac{5}{6}\frac{v^2}{g}$$

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6. Problem 63, p. 280

Solution : The problem requires us to use the conservation of energy twice. On the upper part of the hill, there will be both translational and rotational kinetic energy; on the lower part of the hill, the solid spherical boulder slides on the ice (so there is only translational kinetic energy). If the total height of the hill is h, the boulder rolls through a distance of h/2, and slides through a distance of h/2.

If we consider the first half of the trip, the boulder begins with m g h in GPE, and at h/2, has half of this GPE along with tranlsational + rotational KE. Using results from earlier problems, we know that for rolling without slipping we can write the rotational KE in terms of v:

$$m\,g\,h \;=\; m\,g\,h/2 + \frac{1}{2} m\,v_{h/2}^2 + \frac{1}{2} I\,\omega^2 \;=\; m\,g\,h/2 + \frac{1}{2} m\,v_{h/2}^2 + \frac{1}{2}\left(\frac{2}{5} m\,v_{h/2}^2\right)$$

where $v_{h/2}$ is the speed at the midpoint of the hill. Solving this gives us :

$$v_{h/2}^2 \;=\; \frac{5}{7}\,g\,h$$

Now we consider the second half of the trip. To compute the speed at the bottom, we recognize that the boulder starts with m g h/2 of GPE along with some kinetic energy, and that the boulder slides down the hill (since there is no friction to create rollling). Therefore our energy balance becomes:

$$\frac{1}{2} m\,v_{h/2}^2 + \frac{m\,g\,h}{2} \;=\; \frac{1}{2} m\,v_b^2$$

where v_b is the speed at the bottom. Solving for v_b for an initial height of 50 m:

$$v_b \;=\; \sqrt{v_{h/2}^2 + g\,h} \;=\; \sqrt{\frac{5}{7}\,g\,h + g\,h} \;=\; \sqrt{\frac{12}{7}\,g\,h} \;=\; \sqrt{\frac{12}{7}\left(9.8\,\mathrm{m}/\mathrm{s}^2 \cdot 50\,\mathrm{m}\right)}$$

$$=\; 29\,\mathrm{m}/\mathrm{s}$$