PHYS 111
HOMEWORK #1

Due : 1 Sept. 2016

Homework must be submitted in class on its due date. We will go over these in discussion; no homework can be accepted for a grade once solutions have been posted publicly to the course website. Read the syllabus carefully and make sure you are following the proper format for submitting homework. All problems must show clear and complete work.

This homework assignment is designed to introduce you (or perhaps re-introduce you) to some of the reasoning used in physics, and also to help you review important mathematics for the course.

1. Appendix E at the back of your text provides the astronomical information you will need. Mass density is defined as mass/volume (the volume of a sphere is 4/3 πR^3).

a) Determine the density of the Earth and the sun in kg/m^3

Solution : We begin by writing the equation we will use (the governing equation) :

$$\rho = \frac{\mathrm{M}}{\mathrm{V}} = \frac{\mathrm{M}}{4/3\,\pi\,\mathrm{R}^3} = \frac{3\,\mathrm{M}}{4\,\pi\,\mathrm{R}^3}$$

we substitute numerical values:

$$\rho = \frac{3\left(6\times 10^{24}\ \mathrm{kg}\right)}{4\,\pi\left(6.4\times 10^{6}\ \mathrm{m}\right)^3} = 5500\ \mathrm{kg}/\mathrm{m}^3 \text{ (about 5.5 times denser than water)}$$

b) Imagine a star having the same mass as the sun but a radius 100 times larger than the sun' s. What is the density of this star?

Solution: We first compute the density of the sun:

$$\rho = \frac{3\,\mathrm{M}}{4\,\pi\,\mathrm{R}^3} = \frac{3\left(2\times 10^{30}\ \mathrm{kg}\right)}{4\,\pi\left(7\times 10^{8}\ \mathrm{m}\right)^3} = 1400\ \mathrm{kg}/\mathrm{m}^3$$

Now, for the second part of this question, you could use this equation for density, increase the radius by a factor of 100, and recompute the density of the hypothetical star. It would be better, though, to use proportional reasoning as described below.

We know that the density of an object is inversely proportional to the volume, that is, if the volume increases (while the mass stays the same), the density will decrease. Since the volume depends on the cube of the radius, increasing the radius by a factor of 100 means the volume increases by a factor of 100^3, or 1,000,000. The volume of the larger star is then 1,000,000 times greater than the sun's, meaning that the density of the larger star is 1/millionth of the sun's, or 1.4^{-3} kg/m^3.

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2. The magnitude of centripetal acceleration experienced by an object in circular motion is given by v^2/r where v is the speed and r the radius of the circle. If an object doubles it speed while traveling along the same arc, by what factor does the centripetal acceleration change?

Solution : This is another proportional reasoning problem. If v doubles, then the acceleration increases by a factor of 2^2, or the acceleration quadruples.

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3. The surface gravity on a planet is proportional to M/R^2 where M is the mass of the planet and R is the radius. Consider a planet 100 times as massive as the Earth with a radius 10 times the radius of the Earth. How does the surface gravity on the hypothetical planet compare to the surface gravity on Earth? How does the density of this planet compare to the density of the Earth? (Which planet in the solar system most closely approximates this hypothetical planet?)

Solution : Here we have a proportional reasoning problem with two variables, mass and radius. We are asked to compare values between two planets; mathematically, this means taking ratios. If we call g the surface gravity on a planet, we can compare surface gravities by computing the ratio :

$$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \left(\frac{M_{\text{planet}}}{R^2_{\text{planet}}}\right) \Big/ \left(\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}}\right) = \left(\frac{M_{\text{planet}}}{M_{\text{Earth}}}\right)\left(\frac{R^2_{\text{Earth}}}{R^2_{\text{planet}}}\right)$$

With this ratio in hand, we can substitute numerical values:

$$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \left(\frac{M_{\text{planet}}}{M_{\text{Earth}}}\right)\left(\frac{R^2_{\text{Earth}}}{R^2_{\text{planet}}}\right) = 100 \cdot \left(\frac{1}{10^2}\right) = \frac{100}{100} = 1$$

This means the surface gravities on the two planets are the same. This does not tell you the exact value of the surface gravity, just that the surface gravities on the two planets are equal.

We follow the same procedure to compare the densities:

$$\frac{\rho_{\text{planet}}}{\rho_{\text{Earth}}} = \frac{\left(3\, M_{\text{planet}}/4\,\pi\, R^3_{\text{planet}}\right)}{\left(3\, M_{\text{Earth}}/4\,\pi\, R^3_{\text{Earth}}\right)} = \left(\frac{M_{\text{planet}}}{M_{\text{Earth}}}\right)\left(\frac{R^3_{\text{Earth}}}{R^3_{\text{planet}}}\right) = 100 \cdot \left(\frac{1}{10}\right)^3 = 0.1$$

You can see the power of this technique; by taking ratios, the common factors of $3/4\pi$ cancel out, so you don't have to constantly recompute those values. The final numerical result is also interesting; this value means that the larger planet is 1/10 the density of the Earth. Since we computed the density of the Earth above, we know this planet has a density roughly 0.55 kg/m^3, or a density less than water. The planet Saturn is a close approximation of this hypothetical planet, and in fact, its overall density is less than water's.

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4. If a ball is thrown vertically down from an initial height of H with an initial speed of v, its height above the ground , h, is given by:

$$h = H - v t - \frac{1}{2} g t^2$$

where g is the acceleration of gravity (9.8 m s^{-2}) and t is the time elapsed after the ball is released. If a ball is thrown down with an initial speed of 8m/s from a height of 100m, how long will it take for the ball to reach the ground?

Solution : This problem is essentially solving a quadratic equation. The "physics" part of this problem is to recognize that the statement "when the ball hits the ground" is equivalent to h = 0. Therefore, we want to solve this equation to find the times when h = 0, that is, we want to solve for t in the equation :

$$-4.9 t^2 - 8 t + 100 = 0$$

If you remember from algebra class, a quadratic equation of the form:

$$a x^2 + b x + c = 0$$

where a, b, and c are constants, has the solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4 a c}}{2 a}$$

In this problem, $a = -4.9 \frac{m}{s^2}$, $b = -8 \frac{m}{s}$, $c = 100 m$

so our solutions are:

$$t = \frac{-(-8 m/s) \pm \sqrt{(-8 m/s)^2 - 4 \left(-4.9 m / s^2\right) (100 m)}}{2 \left(-4.9 m / s^2\right)} =$$

$$\frac{8 m/s \pm \sqrt{64 m^2 / s^2 + 1960 m^2 / s^2}}{-9.8 m / s^2} = \frac{8 m/s \pm 45 m/s}{-9.8 m / s^2} = -5.4 s \text{ or } +3.8 s$$

Since we expect the time to be positive, the physically meaningful solution is the positive solution, so the time to reach the ground is 3.8 s after launch.

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5. Solve the following equation for x :

$$x^{2/5} - 3 x^{1/5} + 2 = 0$$

(Do not use calculator or computer algorithms to solve this)

Solution : This may look like a completely impenetrable equation, but it is actually simpler than the one above. We can simplify this problem if we make the substitution u = $x^{1/5}$, then the equation becomes:

$$u^2 - 3\,u + 2 = 0$$

Which factors very simply into:

$$(u - 2)\,(u - 1) \;=\; 0$$

This means that the two solutions are:

$$u \;=\; x^{1/5} \;=\; 2 \;\Rightarrow\; x \;=\; 2^5 \;=\; 32 \text{ and } u \;=\; x^{1/5} \;=\; 1 \;\Rightarrow\; x \;=\; 1^5 \;=\; 5$$

So the two solutions to the original equation are x = 32 and x = 1.

While making the substitution was very helpful, it was not necessary. As long as we recognize this is a quadratic equation in $x^{1/5}$, we could have proceeded as in problem 4:

$$x^{1/5} \;=\; \frac{-(-3) \pm \sqrt{(-3)^2 - 4\,(2)\,(1)}}{2} \;=\; \frac{3 \pm \sqrt{9-8}}{2} \;=\; 2,\,1$$

Again, if $x^{1/5}$=1, $x = 1^5 = 1$ and if $x^{1/5}$=2, $x = 2^5 = 32$ as before.

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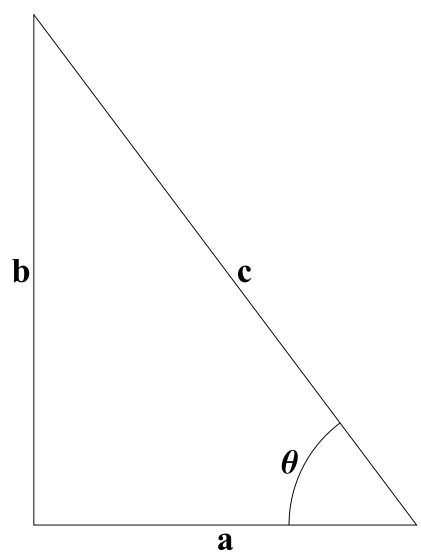
6. Draw a right triangle with sides a and b and hypotenuse c. Use the Pythagorean theorem to show that $\sin^2\theta + \cos^2\theta = 1$.

Solution : Consider the diagram below :



Here the lenghts of the sides are a and b and the hypotenuse has length c. Using the angle chosen for θ, we write the standard definitions of trignometry:

$$\cos\theta \;=\; \frac{a}{c} \qquad\qquad \sin\theta \;=\; \frac{b}{c}$$

Now if we square both trig functions and add them we get:

$$\cos^2\theta + \sin^2\theta = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2}$$

But we know from the Pythagorean theorem that $a^2 + b^2 = c^2$ in a right triangle, so our expression above becomes:

$$\frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1$$

and we have established the identity $\cos^2\theta + \sin^2\theta = 1$ for all angles θ.

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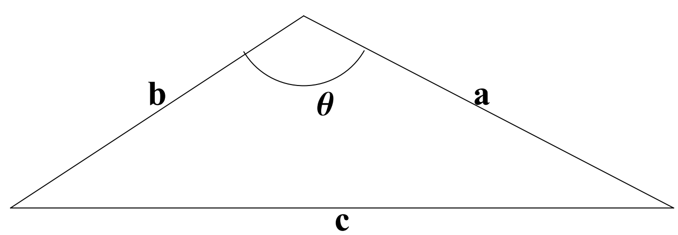
7. Consider a triangle with sides 10, 12, and 19. What is the angle opposite the longest side? What is the angle of the longest side of the sides' lengths are 10, 12 and 24? (If you are puzzled by your answer to this part, think about why you might be getting an odd answer)

Solution : Consider the diagram below :



We want to solve for the angle θ, since it is the angle opposite the longest side, c. Since this is not a right triangle, we must use the law of cosines (see text page 0-15). The law of cosines states:

$$c^2 = a^2 + b^2 - 2\,a\,b\cos\theta$$

where c is the length of one side of a triangle, a and b are the lengths of the other sides, and θ is the angle between a and b (that is, the angle opposite side c). In our example, c = 19, a = 10 and b = 12, so we have:

$$\cos\theta = \frac{c^2 - a^2 - b^2}{-2\,a\,b} = \frac{19^2 - 10^2 - 12^2}{-2\,(10)\,(12)} = \frac{117}{-240} = -0.4875 \Rightarrow \theta = \text{Arccos}(-0.4875) = 119^{\text{o}}$$

If you tried to find the angle for the triangle of sides 10, 12, and 24, you found :

$$\cos\theta = \frac{24^2 - 10^2 - 12^2}{-240} = \frac{-332}{240} \tag{1}$$

which does not yield a solution (since cos θ always has values between -1 and 1). So why don't we get a solution for this situation? Because you can't have a triangle where one side is greater than the sum of the other two sides; you could never "close" such a triangle. Hence, this is not a triangle and there is no meaningful solution.

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8. Kepler' s Third Law of planetary motion relates the orbital period of a planet (P) to the mass of the sun (M), the distance of the planet from the sun (d), and a constant of nature (G). Kepler' s Third Law is usually written as :

$$M\,P^2 = \frac{4\,\pi^2}{G}\,d^3$$

Solve this equation for d. (Meaning rewrite the equation so that d appears alone on one side).

Solution : Rewriting equations to solve for a specific variable is a critical skill in physics. The next two problems give us some practice. Here, we begin by multiplying both sides by $G/4\,\pi^2$:

$$\frac{G}{4\,\pi^2}\cdot M\,P^2 = d^3$$

Now we take the cube root of each side to obtain finally:

$$d = \left(\frac{G\,M\,P^2}{4\,\pi^2}\right)^{1/3}$$

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9. Rewrite equation 11.17 on p. 333 of the text in terms of x.

Solution : Start by writing the equation in question :

$$v_x = \pm\sqrt{\frac{k}{m}}\,\sqrt{A^2 - x^2}$$

And we are asked to solve for x. Since x is inside a square root, we square both sides:

$$v_x^2 = \left(\frac{k}{m}\right)\left(A^2 - x^2\right)$$

Since the square of any real number is positive, we know the right hand side is > 0 and we can eliminate the ± sign. Next we multiply both sides by m/k:

$$\frac{m\,v_x^2}{k} = A^2 - x^2 \Rightarrow x^2 = A^2 - \frac{m\,v_x^2}{k}$$

Finally take the square root of both sides to solve for x:

$$x = \pm\sqrt{A^2 - \frac{m\,v_x^2}{k}}$$

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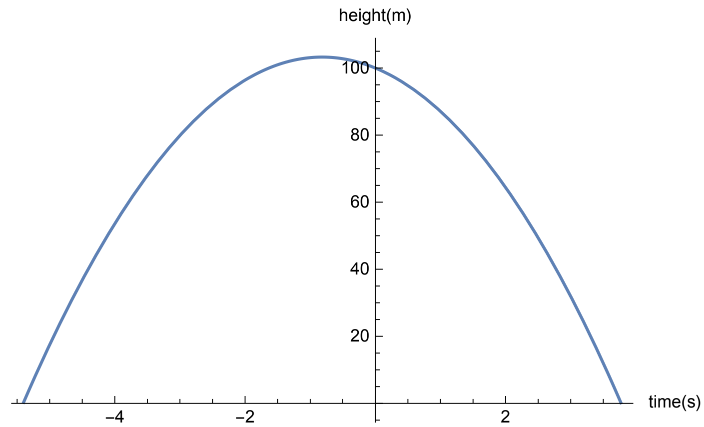
10. In question 4 you solved a quadratic equation, and as you know, there are two solutions to a quadratic equation. The physically meaningful answer gave you a positive value of time; but you should have also found a negative value of time. What is the meaning of the negative answer? In other words, what trajectory would give rise to that result?

Solution : A good way to start analyzing this question is to draw the graph of the function from problem 4 :

$$h(t) = 100 - 8t - 4.9t^2$$



Remember, this is a graph of the height as a function of time; the graph might suggest a parabolic trajectory for the object, but keep in mind the object is moving vertically only.

The portion of the graph for times > 0 shows the situation for the ball thrown down : the ball is at a height of 100 m at t = 0, and reaches the ground when t = 3.77 s.

Now consider the curve for times < 0; this portion of the curve shows us the behavior of the ball if we initially tossed it upward with an initial speed of 8 m/s. It will take this ball 5.4 seconds to reach the ground. So, the negative answer tells us that if a ball was launched upward from the roof with a speed of 8 m/s, it would take 5.4 s to reach the ground.