PHYS111
HOMEWORK #2

Due : 8 Sept. 2016

1. Estimate how many marbles could fit inside a hollow sphere the radius of the Earth. State all assumptions and estimates explicitly, and estimate how many marbles could fit inside an Earth sized sphere. Show all calculations clearly.

Solution : The simplest approach is to determine the volume of a typical marble, and find how many of those would fit inside the Earth. So how do we estimate the size of a marble? After extensive research (i.e., typing in "what is the size of a marble" into Google), I find :

“These balls vary in size.Most commonly, they are about 1 cm (1⁄2 in) in diameter, but they may range from less than 1 mm (1⁄30 in) to over 8 cm (3 in), while some art glass marbles for display purposes are over 30 cm (12 in) wide.Marbles can be used for a variety of games called marbles.” (from the Google)

So, let' s go with the 1 cm diameter marble and we will assume that all marbles will be of the same size. Recall that the volume of a sphere (and we assume spherical marbles) is :

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

If the diameter is 1 cm, the radius is 0.5 cm = 5 10^{-3} meters. We have previously (HW 1 problem 1) used the radius of the Earth, so we can compute:

$$\text{number of marbles that fit into Earth} = \frac{\text{volume of Earth}}{\text{volume / marble}}$$

Now, your instinct might be to compute the volume of a marble, compute the volume of the Earth, and compute the quotient. This will work, but involves too many calculations. This problem provides a good example of the value of inserting numbers only at the very end of the problem :

$\# \text{ marbles} =$

$$\frac{V_{\text{Earth}}}{V_{\text{per marble}}} = \frac{\frac{4}{3}\pi R_E^3}{\frac{4}{3}\pi r_{\text{marble}}^3} = \left(\frac{R_E}{r_{\text{per marble}}}\right)^3 = \left(\frac{6.4\times10^6\ \text{m}}{5\times10^{-3}\ \text{m / marble}}\right)^3 = 2\times10^{27}\ \text{marbles}$$

It’s fine to have stopped at this point. But let’s go a bit further to complete the analysis. This is only an estimate however. If you have ever tried to pack spheres (or sphere like things such as oranges or apples or tennis balls) you know that there is always some space that is unoccupied. One of the great mathematicians of all time, Karl Friedrich Gauss (whose work you will encounter later in this course and especially in Phys 112) showed that the densest packing arrangement of uniform spheres within a larger sphere will occupy a maximum of $\pi / 3 \sqrt{2}$ of the total volume, or about 74% of the total volume. Thus, a more detailed analysis would suggest that the total number of marbles would be 74% of the number determined above.

Are we making any other assumptions? How good is the assumption that all the spheres are of equal size? Could the spheres toward the center compress under the weight of the overlying matter? Yes, we can be certain that the mass of the overlying marbles would squeeze the marbles closer to the center; in fact, approximately 20% of the value of the density of the Earth (which you computed in HW#1) is due to compression of matter. The moral of the problem: No matter how simple a question seems, we can often find deeper and deeper levels of complexity. (Now is a good time to reread section 1.2 in the text).

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2. Experiments show that the period (the time it takes to make one complete cycle) of a pendulum is proportional to the length of the pendulum and the acceleration due to gravity. We can write this statement mathematically as :

$$P \propto L^a g^b$$

where a and b are exponents that you will need to determine (and the symbol "∝" means 'proportional to'). Use the techniques of dimensional analysis established in class to determine the values of a and b.

Solution : Following our procedure from class, we equate the units of period with the units of length and gravity. The units of period are seconds, the unit of length is the meter, and g (the acceleration due to gravity) has units of m s^{-2}. Equating units:

$$s = m^a \left(m\, s^{-2}\right)^b \Rightarrow s^1 = m^a m^b s^{-2b} = m^{a+b} s^{-2b}$$

Now we equate powers. On the left, we have seconds to the first power $\left(s^1\right)$, and this must equal $m^{a+b} s^{-2b}$. Note that mass does not appear on the left; this means that the exponent of mass must equal zero (any number raised to the zeroth power is one), this tells us that:

$$a + b = 0$$

We find the value of b by equating powers of s: $s^1 = s^{-2b} \Rightarrow 1 = -2b \Rightarrow b = -1/2$

If $b = -1/2$, then $a = +1/2$ (since $a + b = 0$). Thus, we have that :

$$P \propto L^{1/2} g^{-1/2} \Rightarrow P \propto \sqrt{L/g}$$

Note : This tells us that the period is proportional to the square root of L/g; this does NOT tell us that $P = \sqrt{L/g}$. In order to turn a proportionality into an equation, we need to know the *constant of proportionality.* In this case, the complete equation is $P = 2\pi \sqrt{L/g}$.

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3. Conceptual Question #12, p. 56 of the text.

Solution : The diagram shows a graph of position vs. time.

In part a) we are asked to rank the magnitude of average velocity for the three different trajectories shown. Average velocity is simply the displacement divided by time. All three particles are at A at the same time, and reach B at the same time. Therefore, they all have the same total displacement in the same total time, meaning all three particles have the same average velocity between points A and B.

In parts b) and c) we are asked to determine the particle with the greatest instantaneous speed at points A and B. Remember that we can determine the instantaneous speed by computing (or estimating) the slope of the tangent line to the curve at a point. At A, particle 3 appears (to me at least) to have the greatest slope, and therefore the greatest speed . The trajectory of particle 1 at A seems almost horizontal, meaning its speed at A is very low. At B, particle 1 has the greatest slope, and therefore the greatest speed.

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4. A person drives a distance D from point A to point B along a straight line at a constant speed of 40 km/hr. On the return trip, the person covers the same distance D at a constant speed of 60 km/hr. What was the average speed for the entire trip? (Hint : The answer is not 50 km/hr)

Solution : We are asked to find the average speed for the entire trip. We know that average speed is defined simply as :

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time}}$$

We know that the total distance traveled for the entire trip is 2D, but we are not told anything explicitly about the total time. We have to figure that out from the information given. Let's do so by considering each leg of the trip separately. It should be clear that the total time is simply equal to the time it takes to get from A to B plus the time to return to A from B. We can write these times as:

$$t_1 = \frac{D}{v_1} \qquad t_2 = \frac{D}{v_2}$$

where t_1 is the time to get from A to B, v_1 is the speed of the trip from A to B, and t_2 and v_2 are the time and speed, respectively, of the return trip (from B to A). (Notice, no numbers until the very end). Then, we can express the average speed as:

$$v_{av} = \frac{2D}{\frac{D}{v_1} + \frac{D}{v_2}}$$

We can add the fractions in the denominator to get:

$$v_{av} = \frac{2D}{\frac{D(v_1+v_2)}{v_1 v_2}} = \frac{2 v_1 v_2}{v_1 + v_2}$$

Notice now that the factors of D cancel out; it doesn't matter how long the trip is. All that matters are the speeds of the outbound and inbound portions. Solving this fraction gives:

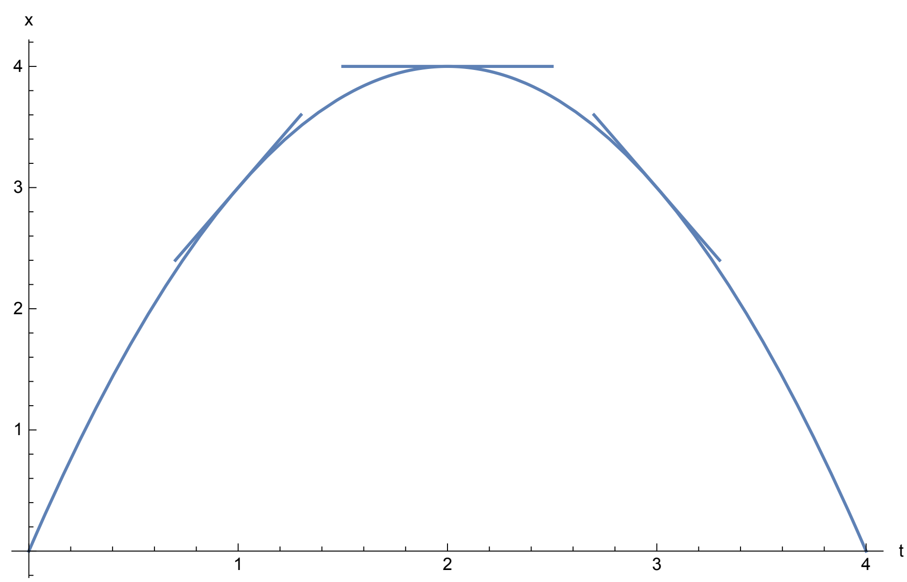
$$v_{av} = \frac{2\, v_1\, v_2}{v_1 + v_2} = \frac{2\,(40\text{ km}/\text{hr} \cdot 60\text{ km}/\text{hr})}{60\text{ km}/\text{hr} + 40\text{ km}/\text{hr}} = 48\text{ km}/\text{hr}$$

6. Conceptual Question #14, p. 57 of text.

Solution : Let' s start with by drawing a graph similar to the one in your text. As we have discussed, of the tangent line at a point on a position vs. time graph will tell you the instantaneous velocity of one dimensional motion.

Look at the tangent line at t = 1; you can see that its slope is positive, indicating the velocity of the particle is positive. This means the particle is moving in the positive x direction. At the apex of the curve, the slope is zero. This means the particle has an instantaneous velocity of zero at this point.

Finally, when t = 3, we see that the slope of the tangent line is negative indicating negative velocity. The graph tells you that the particle is moving toward its starting point of x = 0.



We can use this information and this analysis to answer the questions:

a) Yes, the particle reverses its motion at the apex (at t = 2 on this graph).
b) Yes, the particle returns to the starting point when the graph returns to x = 0, which occurs at t = 4 on this graph.

c) The slopes of the tangent line change, so the velocity is not constant.

d) Yes, the speed is zero where the slope is zero; here, at t = 2.

e) Since the velocity changes, there is an acceleration. (In fact, the acceleration is a = -2).

7. A car travels along a straight line road. Its distance from a stop sign is given as a function of time :

$$x(t) = 2t^2 + 0.25\,t^3$$

where x is distance and is measured in meters, and t is time and is measured in seconds. Calculate the average velocity of the car between the following time intervals:

Solution : In each case, we will use the basic equation of average velocity :

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

and we will use the given equation of motion to determine x(t) for t = 0s, 2s, and 4s.

a) t = 0 s to t = 2 s

$$v_{av} = \frac{x(2) - x(0)}{2\,s - 0\,s} = \left(2\,m/s^2\,(2\,s)^2 + 0.25\,m/s^3\,(2)^3\right) - 0 = \frac{10\,m}{2\,s} = 5\,m/s$$

b) t = 2 s to t = 4 s

$$v_{av} = \frac{x(4) - x(2)}{4\,s - 2\,s} = \frac{2\,m/s^2\,(4\,s)^2 + 0.25\,m/s^3\,(4\,s)^3 - 10\,m}{2\,s} = \frac{38\,m}{2\,s} = 19\,m/s$$

c) t = 0 s to t = 4 s

$$v_{av} = \frac{x(4) - x(0)}{4\,s - 0\,s} = \frac{48\,m}{4\,s} = 12\,m/s$$