PHYS 111
HOMEWORK #4

Due : 22 Sept. 2016

1. A boat travels a distance D along a straight river from point A to point B. In still water, the boat can travel at a constant speed of V_b with respect to the shoreline. The river has a current whose speed is V_R with respect to the shore, and which moves in the direction of A to B.

a) What is the speed of the boat with respect to the shore if it travels from A to B? (5)

Solutions : If the current and boat are moving in the same direction, the velocities will add so that the velocity downcurrent will be :

$$V_{b/s} = V_{b/r} + V_{r/s} = V_b + V_R$$

where the subscripts b, s, and r refer respecitvely to boat, shoreline, and river. Thus, the speed downcurrent is simply the sum of the two speeds.

b) What is the speed of the boat with respect to the shore if it travels from B to A? (5)

When the boat is moving against the current, we should be able to recognize that the boat's speed with respect to the shore will be less since it is fighting against the current. In the formal notation of the book, we can represent this as:

$$V_{b/s} = V_{b/r} + V_{r/s}$$

But the velocities are in opposite directions. Defining the positive direction as returning to the starting point, we can write the speed for the boat in the river for this part of the trip to be:

$$\text{speed}_{b/s} = \text{speed}_{b/r} - \text{speed}_{r/s}$$

c) Show that the time needed for the boat to make a round trip between A and B is given by : (10)

$$t = \frac{2\, V_b\, D}{V_b^2 - V_R^2}$$

Solution : To find an expression for the time of the round trip, we consider each phase of the trip (downstream and upstream) separately. Then, the time for the whole trip is :

$$t_{total} = t_1 + t_2$$

where t_1 is the time to go downstream, and t_2 is the time to go upstream after turning around. We find each individual time by using the simple relationship for constant motion:

$$t = \frac{\text{distance}}{\text{speed}} \Rightarrow t_1 = \frac{D}{V_b + V_R} \quad \text{and } t_2 = \frac{D}{V_b - V_R}$$

Adding fractions gives us:

$$t_{total} = \frac{D}{V_b + V_R} + \frac{D}{V_b - V_R} = \frac{D[(V_b - V_R) + (V_b + V_R)]}{(V_b + V_R)(V_b - V_R)} = \frac{2\, V_b\, D}{V_b^2 - V_R^2} \tag{1}$$

d) Explain the meaning of the answer you obtain in the case where $V_R = V_b$ (5)

If the boat speed upstream is the same as the river current speed downstream, then you will make no progress upstream with respect to the shore. An observer on the shore would see the boat in the same position, neither moving forward nor backward, and you would never return to point A. If you set V_b equal to V_R in eq. (1) above, the denominator goes to zero, indicating an infinite time for the round trip.

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2. Problem 26, page 60 text.

Solution : This problem makes us use feet. Ick. Ok, so the first thing we need to do is convert miles/hr to feet/s. (The result is a handy number to keep in your mind to make easy conversions). Now, let' s make the unit conversion keeping in mind what we know about dimensional analysis.

$$1\, \frac{\text{mi}}{\text{hr}} = 1\, \frac{\text{mi}}{\text{hr}} \left(5280\, \frac{\text{ft}}{\text{mile}}\right)\left(\frac{1}{3600}\, \frac{\text{hr}}{\text{s}}\right) = \frac{5280}{3600}\, \frac{\text{ft}}{\text{s}} = 1.47\, \frac{\text{ft}}{\text{s}}$$

If you are unfamiliar with unit conversions, study this example carefully paying close attention to how the units cancel out.

With this conversion information, we can address the questions of the problem:

$$\text{a) avg acceleration} = \frac{\Delta v}{\Delta t} = \frac{40\text{ mi/hr}\,(1.47\text{ ft/s}) - 60\text{ mi/hr}\,(1.47\text{ ft/s})}{3\text{ s}} = -9.8\text{ ft}\big/\text{s}^2$$

b) We are asked to find the distance traveled and are given initial velocity, final velocity, acceleration and time. We actually have two ways to compute this distance. We can use the time dependent equation of motion :

$$x(t) = x_o + v_{ox}\, t + \frac{1}{2}\, a\, t^2$$

We can set $x_o = 0$ at the point where the brakes were engaged, we know the initial velocity is 60mi/hr = 88ft/s, and the acceleration is - 9.8 ft $\big/ s^2$ (the acceleration is negative since the vehicle is slowing down.) So we find:

$$x(t) = 88.2\text{ ft/s} \cdot 3\text{ s} + \frac{1}{2}\left(-9.8\text{ ft}\big/\text{s}^2\right)(3\text{ s})^2 = 264\text{ ft} - 44\text{ ft} = 220.5\text{ ft}$$

We can also make use of the time independent (meaning there is no term for time in the equation) equation of motion:

$$v_f^2 = v_0^2 + 2\, a\, s \tag{2}$$

where v_f is the final speed, v_o is the initial speed, a is the acceleration and s is the distance traveled. Using values given to us (and making sure we recognize that acceleration is negative:

$$(58.8\,\mathrm{ft/s})^2 = (88\,\mathrm{ft/s})^2 + 2\left(-9.8\,\mathrm{ft}/\mathrm{s}^2\right)\mathrm{s}$$

and we wish to solve for s, the distance traveled while the brakes were being applied. We obtain:

$$\mathrm{s} = \frac{(58.8\,\mathrm{ft/s})^2 - (88.2\,\mathrm{ft/s})^2}{-19.6\,\mathrm{ft}/\mathrm{s}^2} = 220.5\,\mathrm{ft}$$

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3. Problem 28, page 60, text.

Solution : This is a problem in which we have to consider two separate segments of the trip : the free fall phase from the time the cat falls to the time it first makes contact with the floor, and the deceleration phase when the cat' s contact with the floor slows it to rest. Oh, and we also have to do some unit conversions.

a) Finding the velocity at the moment the cat is straightforward. We use Eq. (2) from the previous problem. Here, let' s set down as the positive direction, then the acceleration of gravity is positive.

The distance traveled is also positive. (4 ft = 1.22 meters (see end of problem for conversion)) . If you used 4 ft = 1.22 m for this distance, that's fine, but I think the author wanted you to compute the distance of free fall as 1.22 m - 0.12 m = 1m of free fall. I will use this distance, but will accept either value in your calculations. The cat is dropped, so its initial velocity is zero. We have :

$$\mathrm{v_f^2} = \mathrm{v_0^2} + 2\,\mathrm{a\,s} \Rightarrow \mathrm{v_f^2} = 0 + 2\left(9.8\,\mathrm{m}/\mathrm{s}^2\right)(1\,\mathrm{m}) \Rightarrow \mathrm{v_f} = 4.43\,\mathrm{m/s}$$

Parts b and c): Here we are asked to find the time it takes for the cat to stop and also the acceleration of the cat during its slow down phase. For this part of the problem, our t = 0 occurs when the cat first touches the ground. We know from the previous part that the velocity at this moment is 4.43 m/s. This is a case where the final speed of the first leg of the trip becomes the initial speed of the next leg of the trip.

We can use equation (2) to find the acceleration during this part of the trip, and use the acceleration to find the time to stop :

$$\mathrm{v_f^2} = \mathrm{v_0^2} + 2\,\mathrm{a\,s} \Rightarrow 0 = (4.43\,\mathrm{m/s})^2 + 2\,\mathrm{a}\,(0.12\,\mathrm{m}) \Rightarrow \mathrm{a} = \frac{-(4.43\,\mathrm{m/s})^2}{2\,(0.12\,\mathrm{m})} = -81.8\,\mathrm{m}/\mathrm{s}^2$$

which is the equivalent of $\left(81.8\,m/s^2\right)/\left(9.8\,m/s^2/\mathrm{g}\right) = 8.3$g's

Now we also know that a = Δv/Δt so that we can write:

$$\Delta t = \frac{\Delta v}{a} = \frac{0 - 4.43\ m/s}{-81.8\ m/s^2} = 0.054\ s$$

We can find the stopping time using another method. Since the acceleration is constant, we know that the average velocity for the period when the cat is slowing down is simply:

$$v_{av} = \frac{1}{2}(v_f + v_o) = \frac{1}{2}(0 + 4.43\ m/s) = 2.22\ m/s$$

We also know that for uniform motion, distance = v_{av} t so that t = distance / v_{av} . For these values, that gives us:

$$\text{time} = \frac{0.12\ m}{2.22\ m/s} = 0.054\ s \ \text{ as before.}$$

To put these answers in context, astronauts experience about 3gs during liftoff; fighter pilots can withstand 9gs for a few seconds, but will blackout if the acceleration persists at that rate for more than ten seconds.

Converting feet to meters. Method 1: 4 ft = 4 ft · $\frac{1}{3.28}$m/ft = 1.22 m Method 2: 4 ft = 4 ft · 12 in/ft · 2.54 cm/in · $\frac{1}{100}$m/cm = 48 in · 2.54 cm/in · $\frac{1}{100}$m/cm = 1.22m

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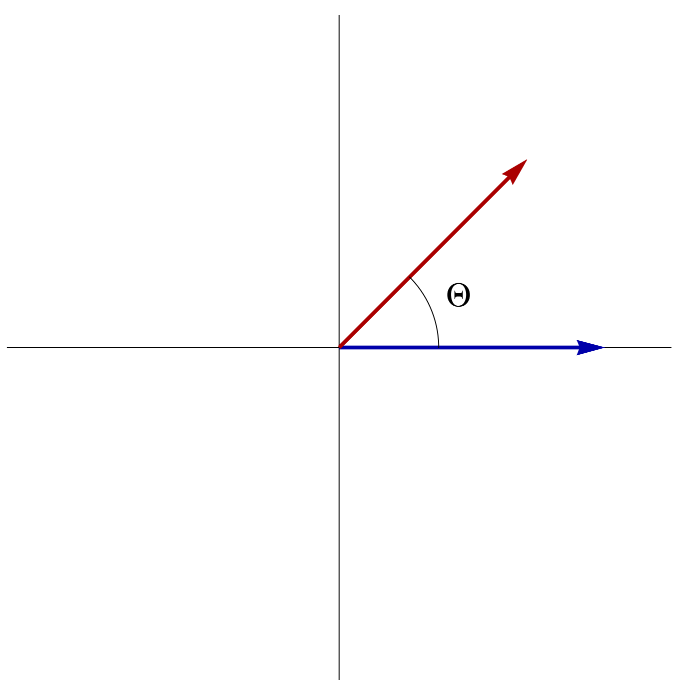
4. A box is located at the origin of a coordinate system. One force of magnitude 720N acts along the positive x axis (as denoted by the blue arrow) and another force pulls on the box with a force of magnitude 360N directed at angle 45^o above the postive x axis. What is the total force on the box, and what angle does the resultant force make with respect to the positive x axis? (“N” stands for Newton, the SI (or MKS) unit of force.)



Solution : This is a straightforward vector addition problem. Let' s say the vector represented by the blue line is vector A and the red line represents vector B. We find the components of A and B :

Since vector A lies along the x axis, we know it has only an x component, so:

$$A_x = 720\,\mathrm{N} \quad A_y = 0$$

We find the components of vector **B**:

$$B_x = 360 \cos 45 = 255\,\mathrm{N} \qquad B_y = 360 \sin 45 = 255\,\mathrm{N}$$

Adding similar components gives us:

$$\text{Total x component} = 720\,\mathrm{N} + 255\,\mathrm{N} = 975\,\mathrm{N}$$

$$\text{Total y component} = 255\,\mathrm{N}$$

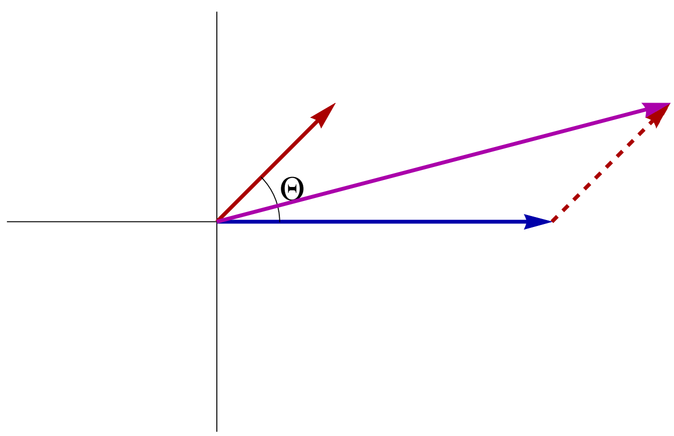
The magnitude of the resultant force acting on the box is then:

$$\text{Magnitude of force} = \sqrt{(975\,N)^2 + (255\,N)^2} = 1008\,\mathrm{N}$$

The resultant force acts in a direction that makes angle θ with respect to the + x axis, which we determine from:

$$\tan\theta = \frac{\text{total y component of force}}{\text{total x component of force}} = \frac{255\,\mathrm{N}}{975\,\mathrm{N}} = 0.26 \Rightarrow \theta = \tan^{-1}(0.26) = 14.6^{\mathrm{o}}$$

Below is shown (to proper scale) the graphical vector addition for these forces. The dashed line is the translated vector **B,** and the magenta line is the resultant vector.:



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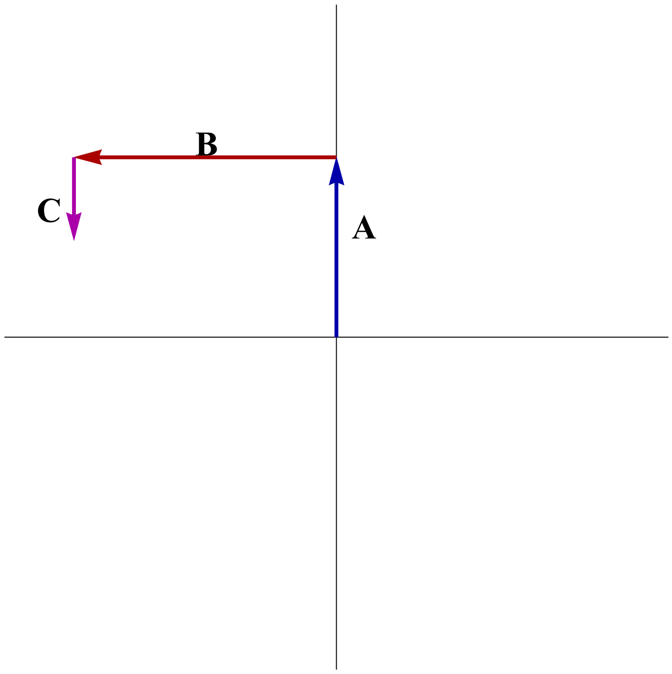
5. Problem 46, p. 25, text.

Solution : First, let' s draw a diagram showing the three parts of the trip :



The procedure in any vector addition problem is to find the x and y components of each vector, sum all the x components, sum all the y components, and find the resultant vector. In this case, that's easy to do since each of the vectors has only one component:

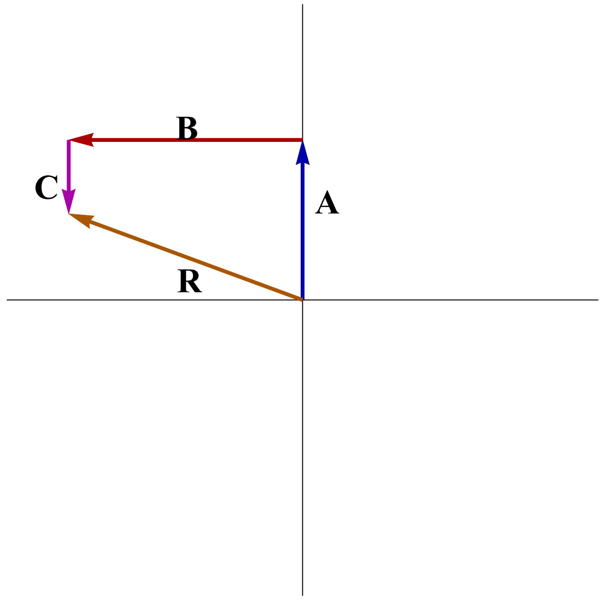
$$A_x = 0 \quad A_y = 3.25\text{ km}$$
$$B_x = -4.75\text{ km} \quad B_y = 0$$
$$C_x = 0 \quad C_y = -1.5\text{ km}$$

Summing all the x components yields

$$R_x = -4.75\text{ km} \quad R_y = +1.75\text{ km}$$

Where **R** refers to the resultant vector (see below):

Out[127]=



We find the magnitude of the resultant vector by using the Pythagorean theorem:

$$|\mathbf{R}| = \sqrt{(-4.75\text{ km})^2 + (1.75\text{ km})^2} = 5.06\text{ km}$$

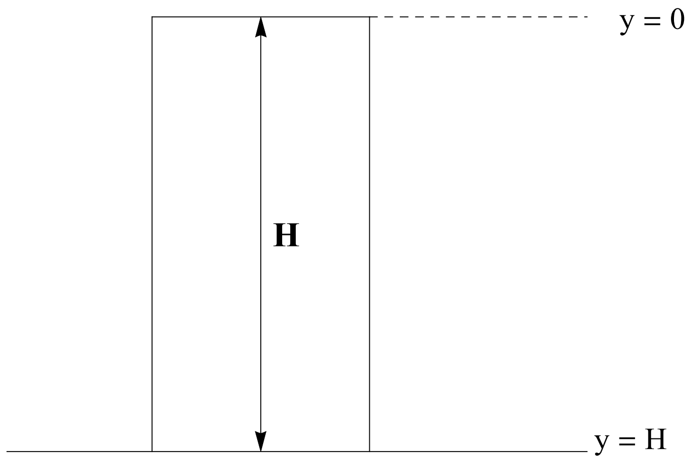
We have a few choices in describing the angle; a simple one is to find the angle between the resultant vector and the negative x axis. We can find this angle from :

$$\tan\theta = \frac{1.75\text{ km}}{4.75\text{ km}} = 0.37 \Rightarrow \theta = \tan^{-1}(0.37) = 20.2^{\text{o}}$$

And we could describe this angle as 20.2^o north of west (or 20.2^o clocwise up from the negative x axis), or alternately we could describe the angle as 69.8^owest of north, or 159.8^o counterclockwise up from the postive x axis. All these phrases refer to the same angle, just choose one that is clearly stated.

6. A ball is dropped from rest from the top of a building of height H. Assuming air resistance is negligible, determine the speed of the ball when it hits the ground and also the time it takes for the ball to hit the ground.

Solution : Let' s consider the diagram below :



For this problem, it is convenient to establish a coordinate system where down is positive, so that the height at the top of the building is y=0 and y = H on the ground. With this choice of coordinate system, both velocity and gravity are positive. We can write our equation of motion as:

$$y(t) = y_o + v_o t + \frac{1}{2} g t^2$$

For this coordinate system, both the initial position and velocity are zero, and we want to know the time when the object hits the ground (where y = H). Thus, we wish to solve:

$$y(t) = H = 0 + 0 + \frac{1}{2} g t^2 \Rightarrow t = \sqrt{2H/g}$$

To find the velocity on impact, we can use the time independent equation of motion:

$$v_f^2 = v_0^2 + 2 a s$$

Here, the initial velocity is 0, the acceleration is + g (since down is the positive direction), and the distance traveled is H (distance traveled = final position - initial position = H - 0). Then we have :

$$v_f^2 = 0 + 2 g H \Rightarrow v_f = \sqrt{2 g H}$$

We could also find the final velocity using the definition of acceleration. We know that acceleration is:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t} \Rightarrow v_f = v_o + a t$$

Recall that the acceleration is g and that the initial velocity is zero, so that:

$$v_f = 0 + g\sqrt{2H/g} = \sqrt{2 g H} \text{ as before}$$