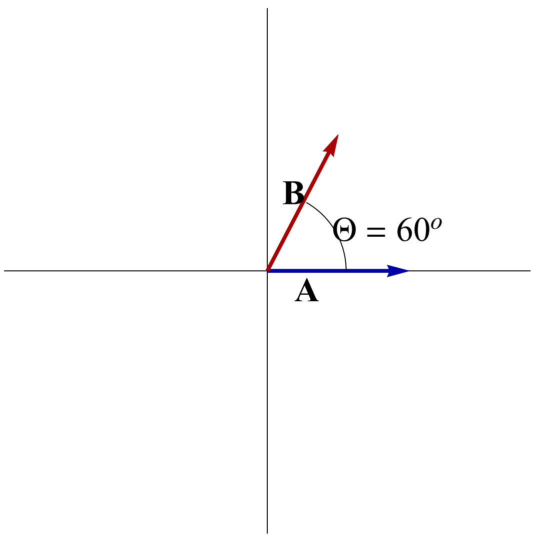
PHYS 111
HOMEWORK #6--Solutions

Due : 18 October 2016

Note : Problems 1 - 5 are from Chapter 4; problems 6 - 8 are from Chapter 5.

1. Problem 2, p. 117, text.

Solution : Let' s define the force of dog A to lie along the x axis, then the force of dog B acts at an angle of 60^o with respect to the x axis:



We find the x and y components of each force:

$$\mathbf{A} = 270\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}}$$

$$\mathbf{B} = 300\cos 60\,\hat{\mathbf{x}} + 300\sin 60\,\hat{\mathbf{y}} = 150\,\hat{\mathbf{x}} + 260\,\hat{\mathbf{y}}$$

We find the resultant vector by adding components:

$$\mathbf{R} = (270\text{ N} + 150\text{ N})\,\hat{\mathbf{x}} + 260\text{ N}\,\hat{\mathbf{y}} = 420\text{ N}\,\hat{\mathbf{x}} + 260\,\hat{\mathbf{y}}$$

the magnitude of this vector is $|\mathbf{R}| = \sqrt{(420\,N)^2 + (260\,N)^2} = 494\text{ N}$

the direction of this vector with respect to the x axis is given by:

$$\tan\theta = \frac{260\text{ N}}{420\text{ N}} \Rightarrow \theta = \tan^{-1}\left(\frac{26}{42}\right) = 31.7^o$$

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2. Problem 5, p. 118, text.

Solution : Refer to the diagram in the text. We decompose each of the three vectors (let' s call A, B, C the vectors in the first, second and third quadrants respectively) :

$$\mathbf{A} = 985 \cos 31\,\hat{\mathbf{x}} + 985 \sin 31\,\hat{\mathbf{y}} = 844\,\hat{\mathbf{x}} + 507\,\hat{\mathbf{y}}$$

$$\mathbf{B} = -785 \sin 32\,\hat{\mathbf{x}} + 785 \cos 32\,\hat{\mathbf{y}} = -416\,\hat{\mathbf{x}} + 666\,\hat{\mathbf{y}}$$

$$\mathbf{C} = -411 \cos 53\,\hat{\mathbf{x}} - 411 \sin 53\,\hat{\mathbf{y}} = -247\,\hat{\mathbf{x}} - 328\,\hat{\mathbf{y}}$$

Summing these to find the resultant vector yields:

$$\mathbf{R} = 181\,\hat{\mathbf{x}} + 845\,\hat{\mathbf{y}}$$

The magnitude of **R** is $\sqrt{(181\,N)^2 + (845\,N)^2} = 864\,\mathrm{N}$

making an angle of $\theta = \tan^{-1}((845\,N)/(181\,N)) = 78^o$ with respect to the x axis.

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3. Problem 8, p. 118, text.

Solution : We use Newton' s second Law :

$$\mathrm{F} = \mathrm{m\,a}$$

we are told the mass of the skater (68.5kg) and are given information to find the acceleration:

$$\mathrm{a} = \frac{\Delta \mathrm{v}}{\Delta \mathrm{t}} = \frac{(0 - 2.40\,\mathrm{m/s})}{3.52\,\mathrm{s}} = -0.68\,\mathrm{m}\big/\mathrm{s}^2$$

(the minus sign indicates that the acceleration is in the direction opposite the motion). Thus the force needed to bring the skater to rest is :

$$\mathrm{F} = \mathrm{m\,a} = 68.5\,\mathrm{kg} \cdot \left(-0.68\,\mathrm{m}\big/\mathrm{s}^2\right) = -46.7\,\mathrm{N}$$

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4. Problem 18, p. 118, text.

Solution : We use the relationship between mass and weight, W = m g where g is the local acceleration due to gravity. The mass of the object is :

$$\mathrm{m} = \frac{\mathrm{W}}{\mathrm{g}} = \frac{44.\,\mathrm{N}}{9.8\,\mathrm{m}\big/\mathrm{s}^2} = 4.5\,\mathrm{kg}$$

On the surface of Io, the mass is the same, and the weight is W = m g = 4.5kg$\cdot$ 1.81 $m\big/s^2$ = 8.1N

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5. Problem 40, p. 119, text.

Solution : To compute the force we will use Newton' s second law, F = m a. We are given the mass but need to compute acceleration. We can find acceleration from the equation :

$$\mathrm{v_f^2} = \mathrm{v_o^2} + 2\,\mathrm{a\,s}$$

since we are told the initial velocity, the final velocity (zero since the car stops), and the distance of travel (0.018m). The most involved part of the problem is converting the speed to m/s, which we do via:

$$45\ \mathrm{km/hr}\ =\ 45\ \mathrm{km/hr}\,(1000\ \mathrm{m/km})\left(\frac{1}{3600}\ \mathrm{hr/s}\right) = 12.5\ \mathrm{m/s}$$

Therefore, we have :

$$0\ =\ (12.5\ \mathrm{m/s})^2 + 2\,a\,(0.018\ \mathrm{m})\ \Rightarrow\ a\ =\ -\frac{(12.5\ \mathrm{m/s})^2}{0.036\ \mathrm{m}} = -4340\ \mathrm{m}\Big/\mathrm{s}^2$$

Then, it is simple to show that $F\ =\ m\,a\ =\ 850\ \mathrm{kg}\left(-4340\ \mathrm{m}\big/\mathrm{s}^2\right)\ =\ -3.69\ 10^6\ \mathrm{N}$

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6. Problem 4, p. 146, text.

Solution : Refer to the diagram in the text. We will first apply Newton' s second law to the 150 N weight, and then to the 72 N weight. Since each weight is at rest (and not accelerating), the sum of all forces acting on each weight is zero.

The forces on the 150 N weight are the weight of the block and the tension in string A. This gives us:

$$\Sigma F_y\ = T_A\ -\ W_A\ =\ 0\ \Rightarrow\ T_A\ =\ W_A\ =\ 150\ \mathrm{N}$$

There are four forces acting on the 72N block; the tension in strings B and C, the weight of the block, and the tension (acting down) in string A. The strings B and C exert tension in both the x and y directions, so we have to decompose those forces.

Summing all forces in the x direction, we get:

$$\Sigma F_x\ =\ T_C \cos 60\ -\ T_B \cos 60\ =\ 0\ \Rightarrow\ T_C\ =\ T_B$$

This information does not tell us the tension B or C, but it does tell us that they are equal in magnitude. Now, summing forces in the y direction:

$$\Sigma F_y\ =\ T_B \sin 60\ +\ T_C \sin 60\ -\ W_B\ -\ T_A\ =\ 0$$

Since $T_B = T_c$, we can write this as:

$$2\,T_B \sin 60\ =\ W_B + T_A\ \Rightarrow\ T_B\ =\ \frac{(W_B + T_A)}{2 \sin 60}\ =\ 128\ \mathrm{N}$$

and this is the tension in both strings B and C.

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7. Problem 13, p. 147, text.

Solution : Refer to the diagram in the text. In both cases, we can apply Newton' s second Law to the weight and show that :

$$\Sigma F_{\text{on weight}}\ =\ T_C\ -\ W\ =\ 0\ \Rightarrow\ T_C\ =\ W\ =\ 250\ \mathrm{N}$$

This is the tension in string C in both cases. Now, we will apply Newton’s second law to the know. Since the knot is not accelerating, all forces must sum to zero.

Part a) we can write the sum of forces in the x and y directions as:

$$\Sigma F_x = T_B \cos 45 - T_A \cos 30 = 0 \Rightarrow T_B \cos 45 = T_A \cos 30$$

$$\text{or } \frac{\sqrt{2}}{2} T_B = \frac{\sqrt{3}}{2} T_A \Rightarrow T_B = \sqrt{3/2}\ T_A$$

$$\Sigma F_y = T_B \sin 45 + T_A \sin 30 - T_C = 0$$

Using the result for T_B in the F_y equation gives us:

$$\sqrt{3/2} \cdot \frac{\sqrt{2}}{2} T_A + \frac{1}{2} T_A = T_C$$

$$\sqrt{3}\ T_A + T_A = 2\,T_c \Rightarrow T_A = \frac{2\,(250\ \text{N})}{\sqrt{3}+1} = 183\ \text{N}$$

$$\text{and } T_B = \sqrt{3/2}\ T_A = \sqrt{3/2}\ (183\ \text{N}) = 224\ \text{N}$$

Part b) : This problem is similar, but we have to be careful to notice that string A pulls down on the knot, and also to be careful in taking components of forces. The horizontal component of tension in string A is (- T_A sin 60), and the vertical component is also negative, (- T_A cos 60). Applying Newton's second law to the knot, we get:

$$\Sigma F_x = T_B \cos 45 - T_A \sin 60 = 0 \Rightarrow \frac{\sqrt{2}}{2} T_B = \frac{\sqrt{3}}{2} T_A \Rightarrow T_B = \sqrt{3/2}\ T_A$$

$$\Sigma F_y = T_B \sin 45 - T_A \cos 60 - T_C = 0$$

This can be rewritten as:

$$\sqrt{3/2}\ \frac{\sqrt{2}}{2} T_A - \frac{T_A}{2} = T_C \Rightarrow \left(\sqrt{3}-1\right) T_A = 2\,T_c$$

or

$$T_A = \frac{2\,T_C}{\sqrt{3}-1} = \frac{500\ \text{N}}{\sqrt{3}-1} = 683\ \text{N}$$

$$\text{and } T_B = \sqrt{3/2}\ (683\ \text{N}) = 837\ \text{N}$$

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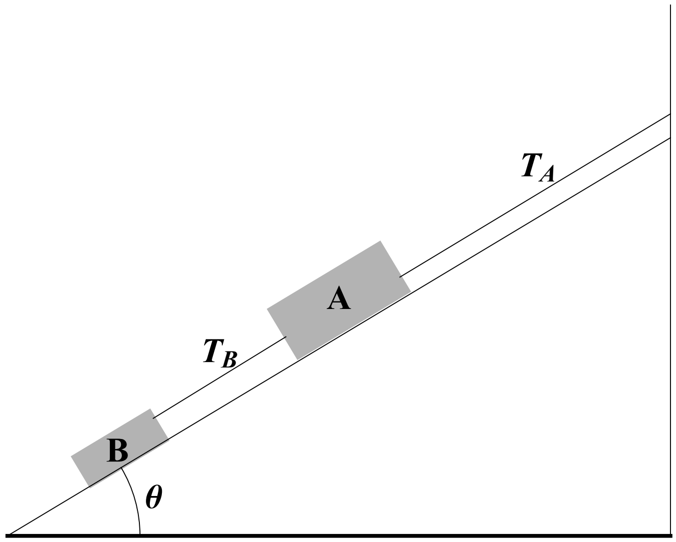
8. Problem 14, p. 147, text.

Solution : Consider the diagram below :



We know from class that the component of gravity acting along the plane is m g sin θ, and the component of gravity acting perpendicular to the plane is m g cos θ.

Since the blocks are held in place by the strings, there is no acceleration in the system, and the sum of forces on each mass must be zero.

Applying Newton's Laws to mass B, we have:

$$\Sigma F_{\text{along plane}} = T_B - m_B\, g \sin\theta = 0 \Rightarrow T_B = m_B\, g \sin\theta$$

$$\Sigma F_{\text{perp plane}} = N_B - m\, g \cos\theta = 0 \Rightarrow N_B = m_B\, g \cos\theta$$

where N_B is the normal force on block B (i.e., the force exerted on B by the plane).

Applying Newton's laws to the block A:

$$\Sigma F_{\text{along plane}} = T_A - m_A\, g \sin\theta - T_B = 0$$

$$\Sigma F_{\text{perp plane}} = N_A - m_A\, g \cos\theta \Rightarrow N_A = m_A\, g \cos\theta$$

Now to find the tension in string A, we solve for T_A and remember that the tension in string B is m_B g sin θ:

$$T_A = m_A\, g \sin\theta + T_B = m_A\, g \sin\theta + m_B\, g \sin\theta = (m_A + m_B)\, g \sin\theta$$

In other words, the tension in string A is equivalent to supporting a single of object of mass (m_A+ m_B) on an incline of angle θ.