FERRIS WHEEL PROBLEM

In addition to this write - up, I have posted two versions of my program solution to the Ferris wheel problem; one which has extensive internal commentary, and one which does not.

Framing the problem

My basic approach to solving this problem was to frame the solution in terms of the person' s angle on the wheel at the moment of stepping off ("launching"). For each angle θ, I calculated the person' s x and y positions on the wheel, the horizontal and vertical components of the person' s velocity, and then used that information to determine how long after launch it would take the person to hit the water.

Knowing that time of flight, it is easy to calculate the x position of the person' s impact (I called this variable xrange). Then, I compared the x position of the person at impact with the x position of the boat (xboat) at the same moment (the time of impact). To find xboat, I had to be careful to use the total elapsed time from t = 0. The total elapsed time equals the time the person was on the wheel rotating to the launch point, plus the time after launch needed to hit the water.

As described in the program commentary, I used two different ways to determine the smallest launch angle that would cause the person to land in the boat. The more straightforward of these was to determine when the landing position of the person (xrange) was within 0.01 meters of the position of the front of the boat (xboat).

Governing Equations

The basic equations used to solve the kinematics aspects of this problem are :

Position of person on the wheel as a function of theta :

$$x0 = R \sin \theta$$
$$y0 = h + R \cos \theta$$

where R is the radius of the wheel, h is the height of the center of the wheel above the water surface, θ is the angle of the radial line to the person with respect to the top of the wheel, and x0 and y0 are the positions of the person on the wheel.

If the person does not jump or contribute to the initial launch velocity, then the person' s initial components of motion equal the velocity components of the edge of the wheel. Written as a function of θ, these are :

$$v_x = \omega R \cos \theta$$

$$v_y = -\omega R \sin\theta$$

where ω is the angular velocity of the wheel.

Thus, the time of launch as a function of θ is found simply from $\theta = \omega$ t ⇒ t = θ/ω; the time to hit the water is found by solving the quadratic :

$$y(T) = 0 = h + R\cos\theta - \omega R \sin\theta T - 1/2\, g\, T^2$$

where we use t to represent the time on the wheel and T to represent the time in the air.

Since the person' s horizontal velocity will not vary during flight (since there are no horizontal fricitonal forces acting), we can find the landing position for the person from :

$$\text{xrange} = \text{x0} + \omega R \cos\theta\, T$$

Here, x0 is the initial x position on the wheel, ω R cos θ is the constant horizontal velocity during flight, and T is the time of the person' s flight.

The position of the boat is then written as :

$$\text{xboat} = 150 - 10\,(T + t)$$

It is important to remember that the boat is traveling at 10 m/s for the entire time the person is in motion, including the time rotating into position on the wheel and also the time in flight in air.

Once we know the time and location of the person' s landing spot in the water, we can ask where the boat is at that time; once the difference between these two position is less than 0.01 m, the do loop terminated and output the position, time, and launch angle of the person. To determine the end of the launch window, I wrote an If statement that would determine when the person' s landing position was greater than 1 m, the length of the boat.

Estimating the launch angle by direct calculations:

I suggested doing a few calculations by hand to see if you could estimate the value of the successful launch angle. Let' s do a few of those here.

First, let' s start with θ = 0.

When the person is at the top of the wheel, the distance above the water is h + R = 110 m. The initial velocity is all horizontal and equal to ω R cos θ = 0.2 rad/s * 30 m * cos 0 = 6 m/s. Since there is no initial vertical velocity, we can calculate T, the time to reach the water from :

$$\text{height} = \frac{1}{2} g\, T^2 \Rightarrow T = \sqrt{2\, h / g} = \sqrt{2 * 110\, \text{m} / 9.81\, \text{m/s}} = 4.74\, \text{s}$$

Since the person falls from the top at t = 0, there is no time spent on the wheel, and we can find the horizontal distance traveled from :

$$\text{xrange} = 6\, \text{m/s} * 4.74\, \text{s} = 28.41\, \text{m}$$

The boat has been in motion for 4.74 s at a speed of 10 m/s, so when the person hits the water, the boat' s distance is :

$$s = \text{xboat} = 150\text{ m} - 10\text{ m/s} * 4.74\text{ s} = 102.6\text{ m}$$

Clearly, the boat and person are not very close at all.

***θ* = *π*/2**

If the person steps off the boat at $\theta = \pi/2$, there is no initial horizontal velocity and all the initial velocity is vertically downward with a magnitude of 6 m/s. The person' s x position is the radius of the wheel, 30 m, and the initial y position is even with the center of the wheel, or 80 m above the water. With these data, we know the person lands directly beneath the edge of the wheel (since there is no horizontal motion). The total time that elapses from t = 0 is the time spent on the wheel rotating into position :

$$\text{time rotating on wheel} = \theta / \omega = \pi / 2\text{ rad} / 0.2\text{ rad/s} = 7.85\text{ s}$$

plus the time in air which is determined by solving the quadratic :

$$y(T) = 0 = 80 - 6T - 1/2\, g\, T^2$$

Solving this by elementary means yields a value of T = 3.47 s. Therefore, the total elapsed time is 7.85 s + 3.47 s = 11.32 s. The boat' s position is then :

$$\text{xboat} = 150\text{ m} - 10\text{ m/s} * 11.32\text{ s} = 36.76\text{ s}$$

and the person and the boat are much closer. Trying one more value of θ :

***θ* = 2 *π*/3 :**

The initial position of the person at the moment of launch :

$$x0 = R\,\text{Sin}\,\theta = 30 \sin 120 = 25.98\text{ m}$$
$$y0 = h + R\cos\theta = 80 + 30\cos 120 = 65\text{ m}$$

Initial vertical velocity components :

$$v_x = \omega R \cos\theta = 0.2 * 30 * \cos 120 = -3\text{ m/s}$$
$$v_y = -\omega R \sin\theta = -0.2 * 30 * \sin 120 = -5.20\text{ m/s}$$

Total elapsed time =

time on wheel + time in air

time on wheel : $\theta / \omega = 2\pi/3 \,/\, 0.2\text{ rad/s} = 10.47\text{ s}$

find time in air from : $y(T) = 0 = 65 - 5.2\,T - 1/2\, g\, T^2 \Rightarrow T = 3.15\text{ s}$

$\text{total time} = 13.62\ \text{s}$

$\text{xrange} = \text{x0} + v_x T = 25.98\ \text{m} + (-3\ \text{m}/\text{s} * 3.15\ \text{s}) = 16.53\ \text{m}$

$\text{xboat} = 150\ \text{m} - 10\ \text{m}/\text{s} * 13.62\ \text{m} = 13.8\ \text{m}$

And we see that the person now lands beyond the boat (and by more than one meter beyond the front of the boat). This tells us that the successful launch window must lie somewhere between $\pi/2$ and $2\pi/3$ radians.