Getting Started With Mathematica

First Steps

This semester we will make use of the software package Mathematica; this package is available on all Loyola networked computers. You can access *Mathematica* by going to the START button in the lower left of your computer screen and:

> Single Click on Start
> Select ALL PROGRAMS
> Select LOYOLA SOFTWARE
> Select MATH AND COMPUTER SCIENCE
> Select MATHEMATICA 7 or MATHEMATICA 8

It may take a little while for *Mathematica* to load.

Basic Calculations and Capabilities

I want you to become familiar with Mathematica during this semester for a number of reasons. First, it is one of the most, if not the most, comprehensive and powerful software packages of its sort. Second, while you may already be familiar with MAPLE or MATLAB, it is always a good idea to know as many different languages or formats as possible

The range of what *Mathematica* can do is quite impressive. It can serve as a simple calculator: For instance, if we wish to find the value of cos 30 degrees, we would enter the input:

    Cos[π / 6]

by simultaneously hitting Shift ENTER (or just the ENTER key on the extended keypad) you will receive the output:

$$\frac{\sqrt{3}}{2}$$

If you want to try more ambitious calculations, we can use *Mathematica* to determine the probability of getting exactly 50 heads in 100 tosses of a fair coin; we would input:
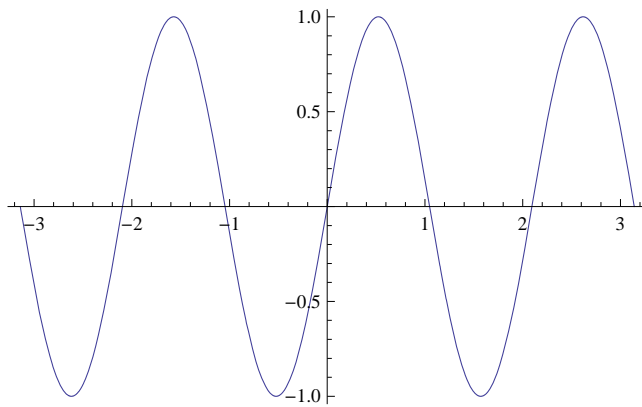
    (100! / (50! × 50!)) (0.5)^100

and recieve the output :

    0.0795892

Mathematica is very useful in creating plots and graphs that can help you understand the nature of a function or solution. For instance, if you want to plot the curve of y=sin(3x) between - π and π , you would input:

```
Plot[Sin[3 x], {x, -π, π}]
```


Suppose you wanted to investigate the behavior of several sine waves at once, you could input :

```
Plot[{Sin[x], Sin[2 x], Sin[3 x]}, {x, -π, π}]
```
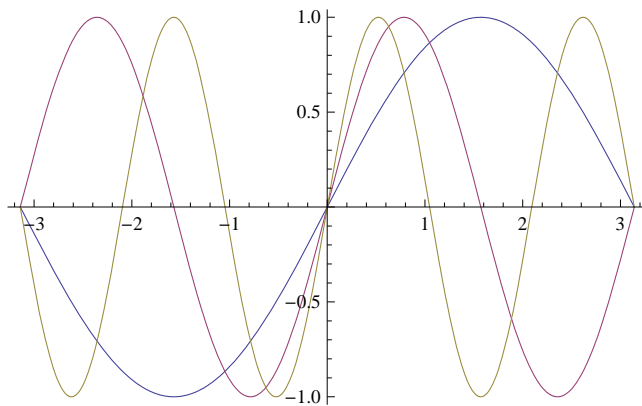

This software also allows us to solve equations, such as $y = 3 x^2 - 2 x + 6 = 0$:

```
Solve[3 x^2 - 2 x + 6 == 0, x]
```

$$\left\{\left\{x \to \frac{1}{3}\left(1 - i\sqrt{17}\right)\right\}, \left\{x \to \frac{1}{3}\left(1 + i\sqrt{17}\right)\right\}\right\}$$

The output shows the two roots of x. We can solve equations analytically; suppose we have a simple Atwood' s machine with masses M and m, we can solve for the tension in the rope and acceleration of the system by entering the equations derived from applying Newton' s Second Law to this system :

```
Solve[{T - m g == m a, T - M g == -M a}, {a, T}]
```

$$\left\{\left\{a \to -\frac{g m - g M}{m + M}, T \to \frac{2 g m M}{m + M}\right\}\right\}$$

here the output line shows in analytic terms the solution to this set of equations.

During the course (especially when we are studying Fourier Series and Transforms), you will be asked to brush off your skills integrating trig functions. One of Mathematica' s most endearing qualities is its ability to integrate almost anything, performing both definite and indefinite integrations. For instance, if we wish to integrate $\tan^3 x$, we would enter:

```
Integrate[Tan[x]^3, x]
```

$$Log[Cos[x]] + \frac{Sec[x]^2}{2}$$

If you want to find the definite integral of the Gaussian function integrated over all space :

```
Integrate[Exp[-x^2], {x, -∞, ∞}]
```

$\sqrt{\pi}$

You are strongly encouraged to use Mathematica to check the results of your work. On many homework assignments, you will be free to use the results of Mathematica rather than carrying out the integrations by hand. (On the first homework, though, I do want you to do the integrals by hand and show your work; however, feel free to use Mathematica as a check.) As we proceed through the semester, you will be able to make more and more use of this software.

Quirks and Cautions

As with any software package, especially one this comprehensive, one can easily feel overwhelmed with all of the possible applications, utilities and opportunities for adding subtelty and nuance to your work. I will do my best not to try to throw too much into any one note, and allow you time to learn the system by doing it; a few simple chunks at a time. I hope you will explore *Mathematica* as much as you can; there is a pretty good online documentation center that will allow you to go into as much depth on any subject as you wish. In this section, I want to mention a few points that you should keep in mind as you begin working with this program.

■ **Functions**

Mathematica has hundreds, maybe thousands of functions. We will use a lot of them this term, most of which will be very familiar to you but some of which may be (and likely will be) brand new. When you call a function, you have to call it exactly as the program expects it to be called. In particular, all functions **must begin with a capital letter.** If you recall our first example, that of finding cos(30), we typed it as:

```
Cos[π / 6]
```

$\frac{\sqrt{3}}{2}$

Mathematica requires that the first letter of all functions be capitalized. (Notice also that we input radians and not degrees). What would happen if we typed ?

```
cos[π / 6]
```

$cos\left[\frac{\pi}{6}\right]$

It outputs exactly what we typed in; by using the lower case, Mathematica assumes that we have defined some new function and is waiting for more instructions.

We show some common function calls along with their outputs:

```
Sin[π / 6]
```

$\frac{1}{2}$

```
Exp[3.74]
```

42.098

```
Log[Exp[3]]
```

3

From the last example, you can see that "Log" refers to natural (base e) logs.

■ **Delimiters: Brackets, Braces and Parentheses**

In Mathematica, brackets, braces and parentheses all have very specific meanings.

Whenever you call a function, you must use **brackets**. For instance, if we try to evaluate:

```
Cos (π / 6)
```

$$\frac{\text{Cos } \pi}{6}$$

We do not get the evaluation that we expect; *Mathematica* requires that we use brackets to evaluate a function: The correct version of this is:

In[82]:= `Cos[π / 6]`

Out[82]= $\dfrac{\sqrt{3}}{2}$

Braces are used when you are denoting lists; you encountered this when we plotted three sine functions on the same graph. In that case, we created a list of the three functions we wished to plot; we also used braces in solving two equations simultaneously (notice there were no braces when we solved just one equation); we use braces in setting up the range over which to plot a function:
$\{x, -\pi, \pi\}$ is the *Mathematica* command meaning "plot over x from $-\pi$ to $+\pi$". Similarly, the braces used in the definite integral $\{x, -\infty, \infty\}$ mean "integrate over x, from $-\infty$ to $+\infty$".

Parentheses are used for grouping as they are in normal mathematical writing; however, in *Mathematica*, only parentheses are recognized as a means of grouping terms to clarify the order of operation

You can read more details about these in the online documentation by typing in "The Four Kinds of Bracketing"

■ **Coefficients and Multiplication**

Let' s say you want to do a simple calculation and multiply 30 times 50. You can do this simply by typing in one number, leaving a space and then typing the next number :

```
30 × 50
```

1500

The "x" that appears was entered automatically by the program when I left a space; the output of 1500 was generated when I hit "Shift Enter"

In this version of *Mathematica*, the program will automatically put a space between a number and a variable. Try to reproduce these steps when you are trying out the program. First, I will set x=4, and then evaluate 2x:

```
x = 4
2 x
```

4

8

If you type the "2" followed by the "x", this version of *Mathematica* will automatically leave a space; older versions of the program did not. Also, notice that we obtained two output lines; the first output line corresponds to the first input line and tells us that we set x = 4. The second output line tells us that when I multiply x times 2, I get 8. (Yes, I know, horribly profound). If I only want the final output, I can suppress the first output by:

```
x = 5;
2 x
```

10

Here, by using a semicolon after the assignment statement I am able to suppress the output. However, if you are inputting an equation symbolically, say F=ma, you will need to leave a space between the variable "m" and the variable "a"; without the space, *Mathematica* will assume you are defining a variable called "ma".

▪ Equal Signs

Mathematica has 3 different types of equal signs; it is important that we learn how to use each one in its proper context.

▪ Immediate Assignment

In traditional mathematics, we use the "=" sign for many different purposes. We make use of this symbol when we want to assign a value to a variable (x=3); we use this sign when we establish a function (f(x)=2x+2); and we use this symbol when we want to solve equations (3x+2y=0). While we use the same symbol for all of them, there is different logic involved in each operation.

There are three different types of equal signs in Mathematica; you have already encountered two of them in this write up. The "=" you are familiar with (and the one used immediately above) is used when you want to make an immediate assignment of a value to a variable. In the examples above, each time we set x equal to a number that number was assigned to x immediately, and will remain assigned as the value of x until we either reassign x or Clear[x]. For instance, if I just type in x and hit "shift enter" :

```
x
```

5

The output is 5, since this is the last value x had from above. If I now evaluate :

```
x ^ 2 − 10 x + 7
```

− 18

The output evaluates the expression with the value of 5 assigned to x.

- **Solving Equations and the Double Equal Sign**

As we saw above, Mathematica is a very useful tool for solving various sorts of equations. When we are solving equations, we make use of a double equal sign "==". As shown above, we can do this for one equation, multiple equations, and can even use this to solve equations symbolically. If we want to solve the general quadratic equation, we would input:

> **Solve[a x^2 + b x + c == 0, x]**
>
> General::ivar : 5 is not a valid variable. ≫
>
> General::ivar : 5 is not a valid variable. ≫
>
> Solve[25 a + 5 b + c == 0, 5]

I made this error on purpose to introduce you to what error messages look like, and to point out an important feature of good programming. Remember just above we said that the statement x = 5 assigned the value of 5 to x as long as we did not re - assign or clear x? Well, even though we wanted to do something else with x, Mathematica remembers and doesn' t understand our Solve statement, thinking "this makes no sense, 5 is not a variable"

So, we clear x and proceed:

> **Clear[x]**
> **Solve[a x^2 + b x + c == 0, x]**
>
> $$\left\{\left\{x \to \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}, \left\{x \to \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}\right\}$$

Note that "Clear[x]" starts with a capital "C", and the argument of the function is in brackets. The output gives us the roots of the quadratic equation as we know and love them. The final "x" after the comma means "solve for x".   Remember, we need to leave spaces between coefficients and variables, if we input:

> **Solve[ax^2 + bx + c == 0, x]**
>
> {{}}

The output we get in this case is Mathematica' s way of saying either there is no solution or it has no idea what we are asking. The difference here is that by leaving no spaces between the a and the x, and the b and the x, it inteprets the cluster of symbols "ax^2" and "bx" to be variables in their own right, and it simply cannot proceed solving for x when the only variables it has are "ax^2" and "bx"

The danger in introducing a new software package is to try to show too much at once; I hope I have not done this. I am not by any means an expert with Mathematica, but have used it enough to think it is a wonderful tool for the sort of work we will do this semester and beyond.

Oh, this set of notes was written in Mathematica.