## **MODELING INFECTIOUS DISEASE**

For the semester modeling project, you will use the techniques of discretizing differential equations we have learned to investigate the behavior of infectious diseases in a population. The model we will use is called the "S-I-R" (or Susceptibility - Infected - Recovered) model.

In this model, the bulk of the population is initially susceptible, that means they have not yet contracted the disease but have the potential to do so. At t = 0, there is a (usually small) number of infected persons, and they will infect a certain number of susceptible persons, causing the (previously) susceptible people to become infected. Assuming the disease does not kill an infected person, each person will recover (after some period of time) and will have recovered, and will have gained resistance to the disease (so that they will not become infected a second time).

We can define our principal parameters as :

S = S (t) = the number of susceptible individuals

I = I(t) = the number of infected individuals

R = R(t) = the number of recovered individuals

This assignment will explore the behavior of infectious diseases using a series of simplifying approximations.

I. The first part of your assignment is to write the first order differential equations to describe ds/dt, di/dt and dr/dt where N is the total population of the city and :

- s (t) = S (t)/N = fraction of the population that is susceptible
- i (t) = I (t)/N = fraction of the population that is infected
- r(t) = R(t)/N =fraction of the population that has recovered.

For this step, do not worry about births, deaths, immigration or emigration, regard the population of the city as fixed. Assume that each infected person will have b contacts per day (b can be greater or less than 1 contact per day depending on your model of the disease); and assume that a fixed fraction k of the infectious group will recover during any given day. If, say, the average period of infection is 3 days, then you should set k = 1/3). You should produce three differential equations, the first of which is :

$$\frac{\mathrm{ds}}{\mathrm{dt}} = -\mathrm{b}\,\mathrm{s}\,(\mathrm{t})\,\mathrm{i}\,(\mathrm{t})$$

Make sure you understand why this equation makes sense, and then use it to determine the differential equations for di/dt and dr/dt. (This part will account for 15% of the total grade)

II. Now, assume that ten infectious people arrive in Chicago (N  $\approx 3 \times 10^6$  people ). Write a short Mathematica program that will model the behavior of s (t), r (t) and i (t). Try at least five different combinations of values for b and k so that you can determine under what conditions the disease effects a large fraction of the Chicago population, and under what conditions the disease is well

contained. (Think in terms of the relationship between b and k). Include a paragraph or two (not more) explaining why the disease behaves differently for different values of b and k. The program you submit should show both your code and graphs for this part. You should compute enough data points until s (t), i (t) and r (t) all reach equilibrium values. Provide a short explanation of why you chose the values of b and k that you did. (This part will account for 50% of the total grade)

III. Use your code to answer the following :

a) Suppose instead of ten infectious arrivals to Chicago, there were 100 or even 1000. For fixed values of b and k, how does changing the value of i (0) effect the peak fraction of infected people in Chicago? How does changing the value of i (0) effect the timing of maximum rate of infection? Make sure you submit graphs to support your answers.

b) For fixed b and k and  $N = 3 \ 10^6$ , determine the total number of infected persons.

c) What is the value of ds/dt + dt/dt? Explain why this is the case. (Part III will account for 20% of the total grade)

IV. Now, extend the model to include births and deaths. For simplicity (and this is what most modelers do) assume that the birth rate and death rate are equal so that the total population remains constant. Amend your differential equations to include births and deaths. (Will the birth rate effect each of s, i, and r (or only a subset of those parameters); will the death rate effect each of s, i, and r or just a subset?). Remember that s, i, and r represent fractions of the population, so make sure your equations have the proper units. (This sections represents 15% of the total grade).

## NOTES :

1) These projects are due no later than 5 pm on Wednesday, April 23. You must submit your code to me as a .nb file using your Loyola account. I must be able to execute your program and reproduce your results. You may submit the other portions of the project either via email or via hard-copy, but I must have all of your work in hand by 5 pm on 23 April.

2) Your grade will be based both on the accuracy of your results and the clarity of your explanations.

3) I urge you to begin working on this as soon as possible. While you have a month to complete this, the last month of the semester is hectic and good programming requires time and care.

4) The work you submit must be the result of your own individual effort. I will give no credit for work that violates this principal.