PROPERTIES OF WAVES DEMONSTRATED THROUGH MATHEMATICA

In class, we began our examination of the properties of waves and ended the period demonstrating these using some simple Mathematica program. First, we considered the wave equation written in familiar form:

\[ y(x, t) = A \sin[2\pi / \lambda(x - vt)] \]

In this equation, the height of the wave is a function of its position (x value) and time, A is the amplitude of the wave, \( \lambda \) the wavelength, v is the wave velocity and t is the time. We can make use of one of my favorite features of Mathematica, the Manipulate function, to verify that this equation generates a traveling wave in the + x direction. We code:

\[\text{In[21]:=} \quad \text{Clear}[a, \lambda, x, v, t] \]
\[y[a_, \lambda_, x_, v_, t_]:= a \sin[(2\pi/\lambda)(x - vt)] \]
\[\text{Manipulate[Plot[y[3, \pi, x, v, t], \{x, -2\pi, 2\pi\}, \{v, 1, 5\}, \{t, 0, 5\}] \]}

If you type this (exactly!) into your Mathematica notebook, you should get this output. If you click on the "+" signs to the right of the sliders, your output will look like:

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By pressing the play button, you will generate a traveling wave.

Let's take a few moments to review the Mathematica code. The first line is a Clear statement. Clear will erase any values for our parameters (in this case, a, λ, x, v, t) that might have been stored in memory. The second line establishes our function y, which is a function of amplitude (a), wavelength (λ), horizontal position (x), velocity (v) and time (t).

Note the format for declaring a function; we use square brackets and underscores (Mathematica calls these "blanks") after each variable. You know that Mathematica has different types of equal signs; to define a function, we use := (a full colon followed by a single equal sign with no intervening space).

The final statement nests the Manipulate and Plot commands. We will learn all about these commands during the semester.

We also considered the properties of sinusoidal waves, and in particular, the properties of the superposition of a sequence of sinusoids. The program below defines the superposition of a number of sine waves:

```mathematica
Clear[wave, ntotal]
wave[n_] := Sum[Sin[n x], {n, 1, ntotal}]
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Here, wave[n] is the sum of ntotal sin waves. So if we set ntotal = 5, wave[5] = sin x + sin 2 x + sin 3 x + sin 4 x + sin 5 x. We can plot the summed wave:
In[30]= Plot[wave[5], {x, -2 π, 2 π}]

Out[30]=

Let's compare this to the clunkier:

In[31]= Plot[Sin[x] + Sin[2 x] + Sin[3 x] + Sin[4 x] + Sin[5 x], {x, -2 π, 2 π}]

Out[31]=

And they match. Please practice your Mathematica skills by typing these codes into a notebook and verifying these results. The more you practice with Mathematica, the more it will become your "native computer language".