VECTOR CALCULUS WITH MATHEMATICA

You should have already read the classnote about gradients. That will remind you about gradients and introduce you to how *Mathematica* can be used to work with vector calculus functions. Remember also that classnote is written in an older version of *Mathematica;* in this document we will show how to call grad,div,curl, Laplacian in the current version of *Mathematica*.

Calling vector calculus functions:

Let's consider the function $f = x\, y^2\, z^3$

We can find the gradient of f via:

```
Clear[f]
f = x y² z³;
Grad[f, {x, y, z}]
```

$\{y^2\, z^3,\ 2\, x\, y\, z^3,\ 3\, x\, y^2\, z^2\}$

We can find the divergence of grad f :

```
Div[Grad[f, {x, y, z}], {x, y, z}]
```

$6\, x\, y^2\, z + 2\, x\, z^3$

And the curl of ∇f:

```
Curl[Grad[f, {x, y, z}], {x, y, z}]
```

$\{0, 0, 0\}$

You should verify all these results by hand. In particular, we will show that the curl of a gradient is always zero.

Contour and Vector Plots

One of the most valuable aspects of *Mathematica* is the ability it gives us to draw visual representations of a vector or scalar field. As shown in the gradient classnote, we consider the function that represents the height of a hill:

$$z = 10\left(2\, x\, y - 3\, x^2 - 4\, y^2 - 18\, x + 28\, y + 12\right)$$

We can show the contour lines of this function:

```
Clear[z]
z = 10 (2 x y - 3 x^2 - 4 y^2 - 18 x + 28 y + 12);
ContourPlot[z, {x, -5, 1}, {y, 0, 6}, ContourLabels → All]
```
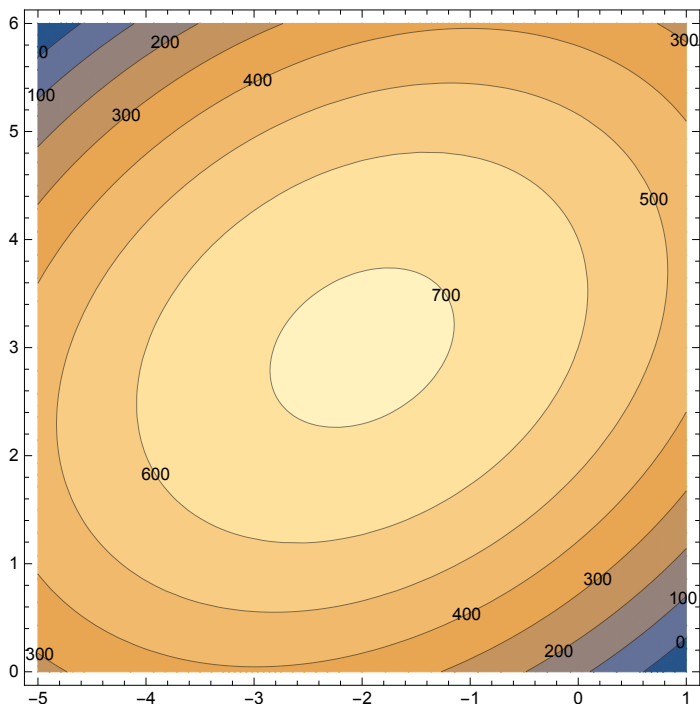

Type this into your own notebook and then scroll your mouse over the contour plot. Notice that the value of the contour line appears when you scroll your mouse over the plot.

Study this plot and determine: a) Where is the hill highest? b) If a placed a rock at (2,3), which way would it roll? c)Which way would a rock placed at (-1,1) roll? c) How could you determine where the height of the hill changed most rapidly? d) In which direction would you have to move to rise (or descend) most rapidly?

Consider now the vector field given by:

$$\mathbf{v} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} \equiv \{x, y\}$$

We can plot this vector field, say, the velocity of a fluid at any point:

```
VectorPlot[{x, y}, {x, -3, 3}, {y, -3, 3}]
```
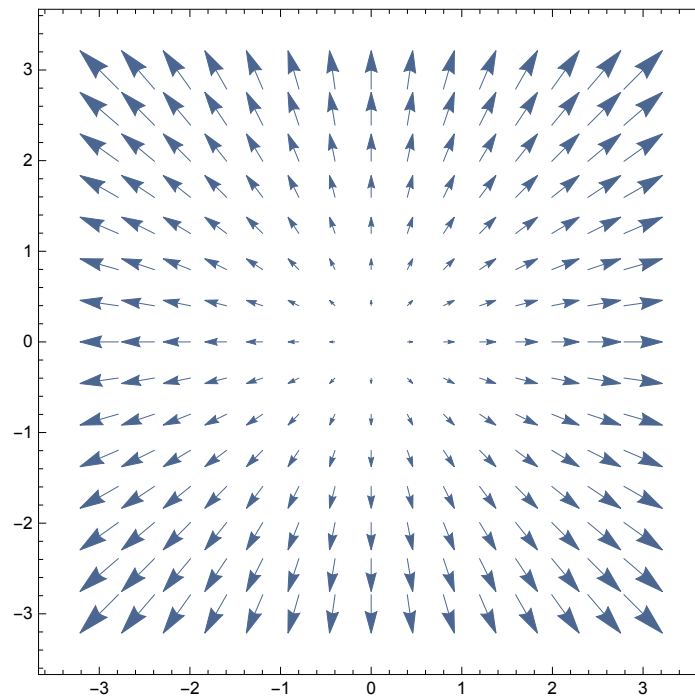

Does this vector field have a divergence? Does it have a curl? What would be the motion of a leaf dropped into a fluid with this velocity profile?