PHYS 301
HOMEWORK #10

Due : 16 April 2012

1. Starting with the Legendre differential equation :

$$\left(1-x^2\right) y'' - 2\,x\,y' + m\,(m+1)\,y = 0$$

Make the substitution :

$$x = \cos\theta$$

and show the equation can be reframed as :

$$\frac{d^2 y}{d\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{d y}{d\theta} + m\,(m+1)\,y = 0$$

This result will be important in solving Laplace' s equation in spherical coordinates. Show clearly and explicitly all steps in this proof.

2. The generating function for Bessel' s functions is :

$$g\,(x, t) = e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n\,(x)\,t^n$$

where the $J_n(x)$ are the Bessel functions of the first kind of order n.

Use the generating function to show that :

a) $J_{n+1} = \frac{2\,n}{x} J_n\,(x) - J_{n-1}\,(x)$

b) $J_n' = \frac{1}{2}\left[J_{n-1}\,(x) - J_{n+1}\,(x)\right]$

3. Expand in a Legendre series (showing the first three non zero terms) :

$$f\,(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

4. Expand in a Legendre series (showing the first three non zero terms) :

$$f\,(x) = \arctan x \quad -1 < x < 1$$

For questions 3 and 4, do all calculations by hand; you may use Mathematica to verify the results of integration, but all other work must be done by hand.

5. Consider three charges lying along the x axis. A charge of - q is at (d, 0), a charge of 2 q is at the origin, and a charge of - q lies at (-d, 0). Use Legendre polynomials to determine the potential due to this arrangement.