PHYS 301
HOMEWORK #4

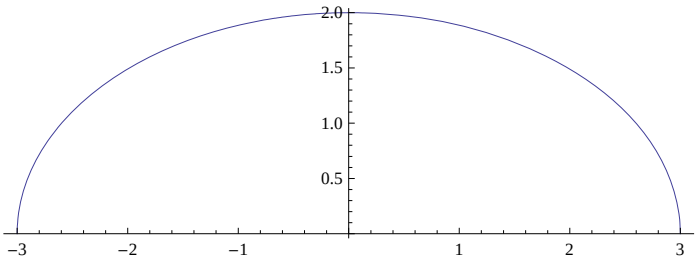
Due : 20 Feb. 2012

1. Consider the function :

$$\mathbf{F} = (e^{x}\cos y - e^{-x}\sin y)\,\hat{\mathbf{x}} + (-e^{x}\sin y + e^{-x}\cos y)\,\hat{\mathbf{y}}$$

Calculate the line integral of this function along the path defined by the upper curve of the ellipse defined by :

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



Hint : Find a contour over which you could apply Stokes' Theorem.

2. Prove that :

$$\int_V (\nabla \times \mathbf{v})\, d\tau = -\int_S \mathbf{v} \times d\mathbf{a}$$

Hint : Replace **v** by (**v** × **c**) in the divergence theorem where **c** is a constant vector.

3. The elliptical cylindrical coordinate system is defined by the transformations equations :

$$x = a\cosh u \cos v$$
$$y = a\sinh u \sin v$$
$$z = z$$

Find the scale factors for this coordinate system. Do this problem by hand; you may use Mathematica to verify your results but you must show all steps by hand. If you are unfamiliar with the hyperbolic functions cosh and sinh, read the classnote on line describing these functions.

4. The spherical polar coordinate system is defined by :

$$x = r\sin\theta\cos\phi$$

$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

Showing all work by hand, find the scale factors for the spherical polar coordinate system and expressions for the unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ in terms of $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$.

5. Read about the function PrimeQ in the Mathematica documentation center and then write a short Mathematica program that will print out the values of all the non prime Fibonacci numbers with n < 200. Your output should include both n and fib[n].