# PHYS 301
# HOMEWORK #7

Due : 23 March 2012

For questions 1 - 4, compute the Fourier trig coefficients and write out the first three non zero terms of each summation. Also write the Fourier series in closed summation form. You must do all integrals for questions 1 and 2 by hand (although you may certainly use Mathematica to check your work); you may do the integrals for questions 3 and 4 either by hand or with Mathematica. Whenever you use Mathematica, make sure you submit your Mathematica results with your homework.

$$1.\ f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$$2.\ f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$$

$$3.\ f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

$$4.\ f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \cos x, & 0 < x, \pi \end{cases}$$

5. Use your solution to problem 1 to show that :

$$\sum_{n,\,odd}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$$

6. Mathematica modelling question:

The motion of a damped oscillator is described by the differential equation :

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + k x = 0$$

where m is the mass of the oscillator, c is the damping constant, and k is the spring constant of the oscillator. As you know from your courses in differential equations, the behavior of an oscillating system will depend on the value of the damping ratio defined as :

$$\zeta = \frac{c}{2 m \omega_0}$$

where $\zeta$ is the damping ratio and $\omega_0$ is the angular frequency of the oscillator and is defined by:

$$\omega_0 = \sqrt{\frac{k}{m}}$$

If the value of $\zeta > 1$, the system is overdamped, and its motion will exponentially decay. If the value of $\zeta = 1$, the system is critically damped and will return to equilibrium in the fastest possible time. If $\zeta < 1$, the system is underdamped and will oscillate while slowly decaying to zero motion.

For this problem, you will investigate the nature of various damped oscillators.

Write a short Mathematica program that will uses discretization techniques to solve the relevant equations to determine the displacement of the oscillator as a function of time. Choose values for m and k for use in this problem.

Using these values of m and k, calculate the value of c that will produce a critically damped oscillator. Vary the value of c and use your code to produce a plot of displacement as a function of time when :

- a) the system is underdamped
- b) the system is critically damped
- c) the system is overdamped

Your output should consist of your code and three plots, each showing the time evolving behavior of the system for the three different values of c. Your plots should show the essential nature of each type of oscillator, so compute enough data points to show the oscillatory or damping nature of each system.

YOU MUST SUBMIT THIS QUESTION TO ME IN A MATHEMATICA NOTEBOOK (.nb file) ELECTRONICALLY. Make sure you clearly note the values of k and m that you have chosen. You may submit questions 1-5 either by hand or via *Mathematica* as usual.

This problem will be worth 50 points; each of problems 1-5 are worth 10 points.