PHYS 301
HOMEWORK #10

Due : 17 April 2013

1. Starting with the Legendre differential equation :

$$\left(1 - x^2\right) y'' - 2\,x\,y' + m\,(m+1)\,y = 0$$

make the substitution x = cos θ and show that this ODE becomes :

$$\frac{d^2 y}{d\theta^2} + \frac{\cos\theta}{\sin\theta}\,\frac{dy}{d\theta} + m\,(m+1)\,y = 0$$

Show explicitly all steps in the proof.

2. The generating function for Hermite polynomials is :

$$g\,(x,\,t) = \mathrm{Exp}\left(2\,x\,t - t^2\right) = \sum_{n=0}^{\infty} \frac{H_n\,(x)\,t^n}{n\,!}$$

where H_n (x) are the Hermite polynomials. Show that this generating function leads to the following recurrence relations :

$$H_{n+1}\,(x) = 2\,x\,H_n\,(x) - 2\,n\,H_{n-1}\,(x)$$

and :

$$H_n{}'\,(x) = 2\,n\,H_{n-1}\,(x) \quad \text{(20 pts)}$$

3. Expand in a Legendre series (showing the first three non - zero terms of the series) :

$$f\,(x) = \begin{cases} 0, & -1 < x < 0 \\ x^2, & 0 < x < 1 \end{cases}$$

4. Expand in a Legendre series (showing the first three non - zero terms of the series) :

$$f\,(x) = \cos x \quad -1 < x < 1$$

5. Consider three charges lying along the x axis. A charge of - q is at (d, 0), a charge of + 2 q is at the origin, and a charge of - q lies at (-d, 0). Use Legendre polynomials to determine the potential due to this arrangement in the x - y plane. See figure in accompanying web page.

6. Boas, problem 1, p. 626.*

7. Boas, problem 2, p. 626.*

*If we do not get to this material in class before the homework is due, students may submit these as extra credit problems.