PHYS 301
HOMEWORK #10-- SOLUTIONS TO PROBLEMS 6 AND 7

6. Boas, problem 1 p. 626. This problem is very similar to the semi - infinte plate solved in class. The difference is that the lower boundary condition is that f (x) = x along the lower edge. Using prior results, we know the general solution will be :

$$T(x, y) = \sum_{n=1}^{\infty} B_n \sin(n\pi x / 10)\, e^{-n\pi y/10}$$

Applying the lower boundary condition gives us :

$$T(x, 0) = \sum_{n=1}^{\infty} B_n \sin(n\pi x / 10) = x$$

We recognize the series as the Fourier sin series for f (x) = x on the interval (-10, 10). We can use our knowledge of Fourier series to determine the coefficients B_n from :

$$B_n = b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin(n\pi x / L)$$

In this case, L = 10 and f (x) = x. Using symmetry (because the integrand is even) we obtain :

$$b_n = \frac{2}{10}\int_0^{10} x \sin(n\pi x / 10)\, dx = -\frac{20\cos(n\pi)}{n\pi} = \frac{-20(-1)^n}{n\pi} = \frac{20(-1)^{n+1}}{n\pi}$$

Substituting this into the general solution we get :

$$T(x, y) = \frac{20}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(n\pi x / 10)\, e^{-n\pi y/10}}{n}$$

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2. Boas, problem 2, p. 626.

This problem is again similar to the semi - infinte plate problem except that for the boundary condition and the width of the plate. In this case, the width of the plate is 20 cm, so the condition that T (20, y) = 0 yields the condition sin (20 k) = 0 $\Rightarrow$ k = n π/20. Our general solution is then :

$$T(x, y) = \sum_{n=1}^{\infty} B_n \sin(n\pi x / 20)\, e^{-n\pi y/20}$$

The lower boundary condition is :

$$T(x, 0) = f(x) = \begin{cases} 0, & 0 < x < 10 \\ 100, & 10 < x < 20 \end{cases}$$

which implies :

$$T(x, 0) = \sum_{n=1}^{\infty} B_n \sin(n \pi x / 20)$$

We solve for the B_n coefficients using the Fourier coefficient :

$$B_n = b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin(n \pi x / L)$$

Here, L = 20 and using symmetry we get :

$$B_n = b_n = \frac{2}{20} \left[\int_0^{10} 0 \cdot dx + \int_{10}^{20} 100 \sin(n \pi x / 20)\, dx \right] = \frac{200}{n \pi} (\mathrm{Cos}(n \pi / 2) - (-1)^n)$$

The general solution is then :

$$T(x, y) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{(\cos(n \pi / 2) - (10)^n) \sin(n \pi x / 20)\, e^{-n \pi y / 20}}{n}$$

The plot below shows that the values computed for B_n will reproduce the lower edge boundary condition :

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Clear[b]
b[n_] := 200 (Cos[n π / 2] - Cos[n π]) / (n π)
Plot[Sum[b[n] Sin[n π x / 20], {n, 1, 71}], {x, 0, 20}]
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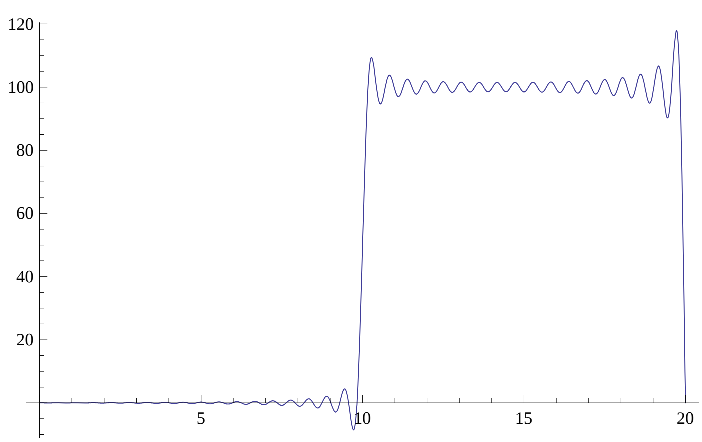
Out[979]=



Which converges, albeit slowly, to the lower edge BC.