PHYS 301
HOMEWORK #2

Due : 30 Jan. 2013

On this homework assignment, you may evaluate integrals either by either direct integration, by employing symmetry arguments, or by citing previous results derived in this course. For instance, if one of the results from homework #1 helps evaluate an integral, you may cite that result and proceed. Display all work and/or provide your reasoning. You may use Mathematica to verify your results, but must submit complete work.

1. For the function :

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 < x < \pi \end{cases}$$

Find expressions for the Fourier coefficients and write the first three non zero terms of each expansion (use the format shown in the answer to problem 2 on page 354 of the text). Do all integrals by hand and show all work.

2. For the function :

$$f(x) = \text{Abs}[x], \; -\pi < x < \pi$$

Find the Fourier coefficients and write out the first three non zero terms of the series expansion.

3. Find the Fourier coefficients and write out the first three non zero terms for :

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin(2x), & 0 < x < \pi \end{cases}$$

4. For $f(x) = \cos \alpha x$, $-\pi < x < \pi$ where α is not an integer, find the Fourier coefficients and first three non - zero terms of the expansion.

5. Use the results of question 4 to show that :

$$\pi \cot \alpha \pi - \frac{1}{\alpha} = 2\alpha \sum_{n=1}^{\infty} \frac{1}{\alpha^2 - n^2}$$

6. Later in the course, we will study a series of orthogonal polynomials on (-1, 1) called Legendre polynomials. The first, second and third order Legendre polynomials are respectively :

$$P_1(x) = x \qquad P_2(x) = \frac{1}{2}\left(3x^2 - 1\right) \qquad P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$$

Show that these 3 Legendre polynomials satisfy orthogonality, namely :

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \begin{cases} 0, & m \neq n \\ \neq 0, & m = n \end{cases}$$

If we wish to normalize the Legendre polynomials, i.e., :

$$c \int_{-1}^{1} P_m(x)\, P_m(x)\, dx = 1,$$

deduce the expression for the factor c which will satisfy orthonormality.