PHYS 301
HOMEWORK #4

Due : 15 February 2013

On this homework, you may use Mathematica for help in evaluation of indefinite integrals only. You must do all other work (including evaluation of coefficients) by hand. If you use Mathematica, please submit your output with your homework, and make sure your name is on your homework. As with all homeworks this semester, make sure your solutions show your work clearly.

1. In class, we outlined the solution to the wave equation as illustrated in problem 9 - 23 of Chapter 7. Compute the Fourier coefficients for this problem and verify that :

$$a_0 = a_n = 0$$

$$b_n = \begin{cases} 0, & \text{n even} \\ 8\,h \big/ \left(n^2 \pi^2\right), & n = 1, 5, 9, \ldots \\ -8\,h \big/ \left(n^2 \pi^2\right), & n = 3, 7, 11, \ldots \end{cases}$$

2. Consider the function :

$$f(x) = \begin{cases} 10, & 0 < x < 10 \\ 20, & 10 < x < 20 \end{cases}$$

Find the appropriate Fourier series for this function if you a) extend this function to make it an odd function, and b) extend it to make an even function. You will see boundary conditions like this again in our study of partial differential equations (section 13.2).

3. Find the Fourier coefficients for f (x) = x on (-1, 1), and use these values with Parseval' s theorem to determine :

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

4. Find the Fourier coefficients for f (x) = x^2 on (-1/2, 1/2) and use these values with Parseval's theorem to determine:

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

5. Consider a vector A that is a function of time whose magnitude is constant. Show that the vector is perpendicular to its time derivative, i.e.,

$$\frac{d\mathbf{A}(t)}{dt} \cdot \mathbf{A}(t) = 0$$

(It is not correct to say that since the magnitude is a constant, d**A**/dt is always zero.)
Describe a physical situation in which the time derivative of a vector is perpendicular to the vector.

6. Evaluate :

a) $\delta_{ij}\,\delta_{jk}\,\delta_{km}\,\delta_{im}$ b) $\epsilon_{ijk}\,\delta_{jk}$

Where the δ are Kronecker deltas and ϵ is the Levi - Civita permutation tensor.

7. Write short Mathematica programs that will find the product of the first 20 Fibonacci numbers. One program should compute this using a Do loop, another using a For statement, and the third using a While statement.