PHYS 301
HOMEWORK #6

Due : 13 March 2013

In this homework set, **r** represents the position vector and r represents the scalar magnitude of the position vector. In questions 2, 3, 4 and 6, use Einstein summation notation (no credit will be given for proofs using term-by-term component expansion). You may use *Mathematica* to verify your answers, but you must show complete work for all problems.

1. Evaluate $\nabla \cdot \left(r^3 \, \mathbf{r}\right)$ (your answer will be in terms of r)

2. Prove $\nabla\left(\frac{f}{g}\right) = \frac{g \nabla f - f \nabla g}{g^2}$ where f and g are scalars.

3. Prove $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\,\mathbf{A} - \mathbf{B}\,(\nabla \cdot \mathbf{A}) - (\mathbf{A} \cdot \nabla)\,\mathbf{B} + \mathbf{A}\,(\nabla \cdot \mathbf{B})$

4. A vector field is called irrotational if its curl is zero. A vector field is called solenoidal if its divergence is zero. If **A** and **B** are irrotational, prove that $\mathbf{A} \times \mathbf{B}$ is solenoidal.

5. Evaluate $\nabla^2\left[\nabla \cdot \left(\mathbf{r} / r^2\right)\right]$

6. Prove curl grad $\phi = 0$ for all scalar fields ϕ (i.e., $\nabla \times \nabla\phi = 0$).