PHYS 301
HOMEWORK #8

Due : 27 March 2013

For questions 1 and 2, you may use Mathematica to verify intermediate results but you must show all work by hand.

1. The transformation equations for parabolic coordinates are :

$$x = u\,v\cos\phi$$
$$y = u\,v\sin\phi$$
$$z = \frac{1}{2}\left(u^2 - v^2\right)$$

Verify that this is an orthogonal transformation (there are a number of ways to do this, you only need to show this once). Find expressions for the scale factors and the unit vectors $\hat{\boldsymbol{u}}$, $\hat{\boldsymbol{v}}$, $\hat{\boldsymbol{\phi}}$.

2. The transformation equations for the spherical coordinate system are :

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

where r is the distance from the origin, θ is the polar angle (measured down from the north pole) and ϕ is the azimuthal angle.

a) Find the scale factors and unit vectors for the spherical coordinate system. (20)
b) Express the position vector completely in terms of spherical polar coordinates. (10)
c) Find the expressions for velocity and acceleration in spherical polar coordinates. (30)

3. Use numerical methods to solve the differential equation :

$$\frac{dy}{dx} = y \text{ subject to the condition } y(0) = 1$$

Write a short Mathematica program to solve this differential equation (using Euler' s method). Your output should include your program and a plot of your tabulated results from x = 0 to x = 3. (Do not use DSolve or NDSolve or any other Mathematica library function that directly solves differential equations). (25)