1. Consider scalar potentials of the form :

\[ V = c r^n \]

where \( c \) and \( n \) are constants and \( r \) is distance from the origin. For which values of \( n \) will this form of a scalar potential satisfy Laplace’ s equation? Comment on the likely usefulness of Laplace’ s equation in problems involving the gravitational and electrostatic forces.

2. Consider a force given by :

\[ \mathbf{F} = e^x \sin y \, \mathbf{\hat{x}} + \left( e^x \cos y - z^2 \sin y \right) \mathbf{\hat{y}} + 2 z \cos y \, \mathbf{\hat{z}} \]

Determine if this is a conservative force. If it is, find the scalar potential from which it is derived, and determine the work done by the force in moving a particle from the origin to \((1, 2, 3)\).

Use series solutions techniques to solve the differential equations :

3. \( x y' = 3 y + 3 \)

4. \( y'' - 2 y' + 3 y = 0 \)

5. \( y'' + y = 0 \)

6. In lab we showed how to use discretization methods and Euler’ s method to solve trajectory problems in two dimensions in the absence of friction. For this problem, we will now add linear friction, meaning that the projectile will experience frictional force that varies as :

\[ \mathbf{F} = -k \, \mathbf{v} \]

where \( k \) is a constant and \( \mathbf{v} \) is the vector velocity. Write a short Mathematica program to determine the trajectory of a projectile whose launch speed is 30 m/s at an angle of 45 degrees. Assume the value of \( k \) is 0.5. Your program should explicitly calculate time of flight, maximum height, and range. Your output should include a plot of the trajectory of the projectile in the case where there is friction and compare it on the same set of axes to a graph of the trajectory if \( k = 0 \). To receive full credit for this problem, you will determine the values of range, max ht and time of flight completely within the program (and will not need to do any visual inspection of data or do multiple executions of the program to determine these parameters). (40 pts)