PHYS 301
HOMEWORK #1

Due : Friday, 17 Jan. 2014

All assignments must be turned in at the beginning of class on the day they are due. All answers must be accompanied by complete and clear work.

1. Find the average value of $\sin^3 x$ on the interval $[0, \pi]$. Show how you solve this integral ; you may use Mathematica or integral tables to check your work, but you must explicitly show how you integrate this function).

2. Using power series as appropriate, show that :

$$e^{i x} = \cos x + i \sin x$$

where i is the imaginary number ($i = \sqrt{-1}$). You need not compute the appropriate Taylor series, you may quote the well known results.

3. Consider the differential equation :

$$\left(1 - x^2\right) \frac{d^2 y}{dx^2} - 2 x \frac{dy}{dx} + k y = 0$$

Make the substitution $x = \cos \theta$ and rewrite this differential equation in terms of $y(\theta)$. (Do not attempt to solve this differential equation; we will spend considerable time investigating the properties of this equation later in the course).

4. The transformation equations for Cartesian to cylindrical polar coordinates are :

$$x = \rho \cos \phi \quad \text{and } y = \rho \sin \phi$$

where ρ is the distance from the origin and ϕ is the azimuthal angle (the angle measured counter-clockwise upward from the + x axis). Write the total derivatives for dx and dy in terms of ρ and ϕ.

5. Consider a particle whose distance from the origin is given by :

$$s = 3 \sin (2 t + \pi / 8) + 3 \sin (2 t - \pi / 8)$$

where s is distance and t is time. What are the amplitude and period of this function?