

PHYS 301

HOMEWORK #11

Due : 4 April 2014

1. Consider the transformation :

$$x = u v \cos \phi$$

$$y = u v \sin \phi$$

$$z = \frac{1}{2} (u^2 - v^2)$$

Show that the (u, v, ϕ) coordinate system is orthogonal, and find the scale factors and unit vectors.

2. Refer to the webpage : http://en.wikipedia.org/wiki/File:Gimbal_3_axes_rotation.gif

Assume the {pink, blue, green} rings have radii {P, B, G} respectively. Consider a point fixed on either the green or blue ring. Use the expression for acceleration in spherical polar coordinates to determine an expression for the acceleration of this point. (10) Now determine an expression for the acceleration of a particle fixed to the pink ring.(10) Assume all angular velocities are constant. Explain clearly (but briefly) which terms you set equal to zero and why you can do so. What is the acceleration of a point on the North Pole of the pink ring? (5) How long could you stare at the figure before you got dizzy?(2)

3. For what, if any values of n , will the function

$$V = r^n \cos \theta$$

satisfy Laplace's equation in spherical coordinates?

Use series solutions techniques to find the recursion relations for the following differential equations, and write out the first three non - zero terms of the solution (and the first three non - zero terms of each branch if there are more than one branch). You may use Mathematica to verify solutions, but to receive credit you must submit your own solutions by hand.

3. $y'' + y = 0$

4. $y'' + x y = 0$

5. $y'' - 2x y' - 2y = 0$

6. $y'' - x^2 y' - y = 0$