PHYS 301

HOMEWORK #8

Due : 19 March 2014

1. Consider the vector force function :

$$\mathbf{F} = y^2 z^3 \hat{\mathbf{x}} + 2 x y z^3 \hat{\mathbf{y}} + 3 x y^2 z^2 \hat{\mathbf{z}}$$

a) Determine whether the force is conservative. (5)

b) If the force is conservative, determine the scalar potential which generates it. (10)

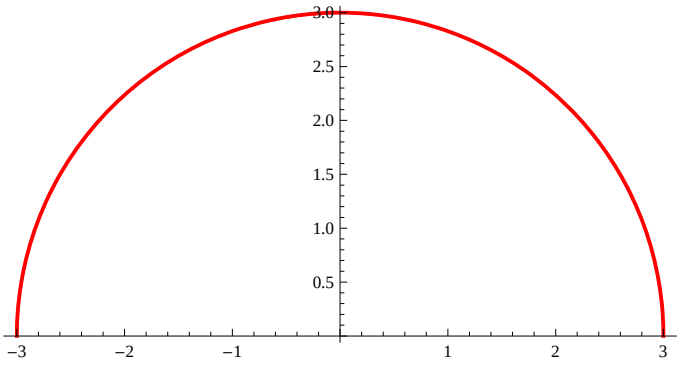
c) Compute the work done by this force if it acts between (0, 0, 0) and (1, 2, 3). (10)

2. Consider the force : (10 pts for each part)

$$\mathbf{F} = e^x \cos y \hat{\mathbf{x}} - e^x \sin y \hat{\mathbf{y}}$$

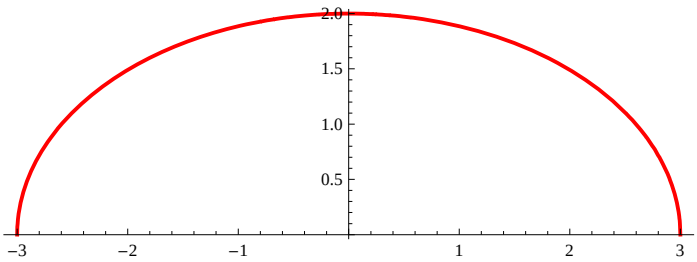
Compute the work done by this force in acting along the path:

a) the upper semicircle of the circle of radius 3 centered on the origin (going from - 3 to 3) :



b) The upper half of the ellipse (also going from - 3 to 3) :

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



(Hint : ***Think*** before you feel the need to solve very elaborate integrals)

3. If **r** is the position vector, compute

$$\int_S \mathbf{r} \cdot \mathbf{n}\, da$$

where the surface is : a) the surface of a sphere of radius 3 centered on the origin, and b) a cube of length L. (10 pts each part)

4. Prove that :

$$\int_V (\nabla \times \mathbf{v})\, d\tau = -\int_S \mathbf{v} \times \mathbf{da}$$

Hint : Replace **v** by **(v × c)** in the divergence theorem, where c is a constant vector.