In questions 1-5, use summation notation to show that:

1. \( \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \)

2. \( \nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A} (\nabla \cdot \mathbf{B}) - \mathbf{B} (\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B} \)

3. \( \nabla \times (f \mathbf{A}) = f (\nabla \times \mathbf{A}) - \mathbf{A} \times \nabla f \) where \( \mathbf{A} \) is a vector and \( f \) is a scalar.

4. Show that \( \nabla \times (\nabla \phi) = 0 \) always where \( \phi \) is a scalar field.

(In this case, it might help you to use determinants to compute this cross product to help you understand exactly why this identity is zero. Then use summation notation to prove the identity.)

5. \( \nabla (\phi \psi) = \phi \nabla \psi + \psi \nabla \phi \) where \( \phi \) and \( \psi \) are both scalars.