HOMEWORK #8-- Solutions

In[176]:=

(* No friction case *)
Clear[x, y, vx, vy, h, g, nterms, nofrictioncase]
x[0] = 0; y[0] = 0; g = 9.8; h = 0.01; v0 = 40; \theta = 2 \pi / 9;
vx[0] = v0 Cos[\theta]; vy[0] = v0 Sin[\theta];
v[y_] := vy[n] = vy[n - 1] - gh
vx[n_] := vx[n] = vx[n - 1]
y[n_] := y[n] = y[n - 1] + vy[n - 1] h
x[n_] := x[n] = x[n - 1] + vx[n - 1] h

nterms = Catch[Do[If[y[n] < 0, Throw[n - 1]], {n, 1000}]];
nofrictioncase = ListPlot[Table[{x[n], y[n]}, {n, nterms}]]
maxht = Catch[Do[If[vy[n] < 0, Throw[y[n - 1]]], {n, nterms}]];
Print["Maximum height evaluated from velocity condition = ", maxht, " meters"]
(* maxht determines maximum height by determining where vy<0 *)
maxht2 = Catch[Do[If[y[n] < y[n - 1], Throw[y[n - 1]]], {n, nterms}]];
(* maxht2 determines max ht by finding where y[n] values begin to decrease *)
Print["Max height evaluated from position condition = ", maxht2, " meters"]
Print["Time of flight = ", nterms h, " seconds"]
Print["Range = ", x[nterms], " meters"]
Print["


Out[184]=

Maximum height evaluated from velocity condition = 33.857 meters
Max height evaluated from position condition = 33.8573 meters
Time of flight = 5.25 seconds
Range = 160.869 meters
In[210]:=

(* With friction *)
Clear[axf, ayf, vxf, vyf, xf, yf, ntermsf, h, k, g, frictioncase, vxgraph, vygraph]
xf[0] = 0; yf[0] = 0; v0 = 40; \[Theta] = 2 \pi / 9; vxf[0] = v0 Cos[\[Theta]];
vyf[0] = v0 Sin[\[Theta]]; h = 0.01; k = 0.5; g = 9.8; m = 1;
axf[vxf_] := axf[vxf] = -(k/m) vxf
ayf[vyf_] := ayf[vyf] = -(k/m) vyf - g
vyf[n_] := vyf[n] = vyf[n-1] + ayf[vyf[n-1]] h
vxf[n_] := vxf[n] = vxf[n-1] + axf[vxf[n-1]] h
yf[n_] := yf[n] = yf[n-1] + h vyf[n-1]
ntermsf = Catch[Do[If[yf[n] < 0, Throw[n-1]], {n, 1000}]]
frictioncase = ListPlot[Table[{xf[n], yf[n]}, {n, ntermsf}]]
maxht = Catch[Do[If[vyf[n] < 0, Throw[yf[n-1]]], {n, 1000}]]
Print["time of flight = ", ntermsf h, " seconds"]
Print["Max ht = ", maxht, " meters"]
Print["Range = ", xf[ntermsf], " meters"]

vxgraph = ListPlot[Table[{n h, vxf[n]}, {n, ntermsf}], PlotLabel -> "vx vs, time"]
(* vxgraph plots the velocity in the x direction as a function of time *)
Print[" "]
Print[" "]
vygraph = ListPlot[Table[{n h, vyf[n]}, {n, ntermsf}], PlotLabel -> "vy vs time"]
(* vygraph plots the velocity in the y direction as a function of time *)

Out[219]=

time of flight = 4. seconds
Max ht = 18.6543 meters
Range = 53.0312 meters
\textit{vx vs, time}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{vx_vs_time.png}
\caption{vx vs time graph}
\end{figure}

\textit{vy vs time}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{vy_vs_time.png}
\caption{vy vs time graph}
\end{figure}

\textbf{In[55]=}
\texttt{Show[nofrictioncase, frictioncase, PlotLabel \to \"Comparison of trajectories\"]}
\texttt{Print[\" \"]}
\texttt{Print[\" \"]}

\textbf{Out[55]=}
\texttt{Comparison of trajectories}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{comparison trajectories.png}
\caption{Comparison of trajectories graph}
\end{figure}
Now, let's look a bit more at the graphs for velocity as a function of time. For the case of linear friction that we have here, we can solve these equations (and the equations for x(t) and y(t)) analytically. Let's start with the differential equation for velocity in the x direction. Using Newton's second law in the x direction, we have:

$$m \frac{dv_x}{dt} = -kv_x$$

Using elementary techniques of solving first order differential equations, you obtain:

$$v_x(t) = v_x(0)e^{-kt/m}$$

where $v_x(0)$ is the initial x velocity, which here is $40 \cos(40^\circ)$. Let's superimpose plots of $v_x(t)$ computed using this equation with the tabulated values of $v_x$ determined from our Euler's method code:

```
In[58]:= vxtheoretical = Plot[v0 Cos[\[Theta]] Exp[-k t / m], {t, 0, 4}];
Show[vxtheoretical, vxgraph]
```

And you can see excellent agreement. We do the same below and compare the tabulated vs. theoretical results for the y component of velocity:

```
In[67]:= vytheoretical = Plot[(m / k) ((k v0 + m g) / m) Exp[-k t / m] - g, {t, 0, 4}];
Show[vytheoretical, vygraph]
```