

SIMULATING PROJECTILE MOTION WITH LINEAR AIR RESISTANCE

In class we have studied fairly extensively the motion of a projectile in a resistive medium. For an object launched from level ground, we can write Newton's second law as:

$$m \ddot{x} = -k \dot{x}$$

$$m \ddot{y} = -mg -k \dot{y}$$

Integrating these equations twice and applying initial conditions yields the following equations of motion:

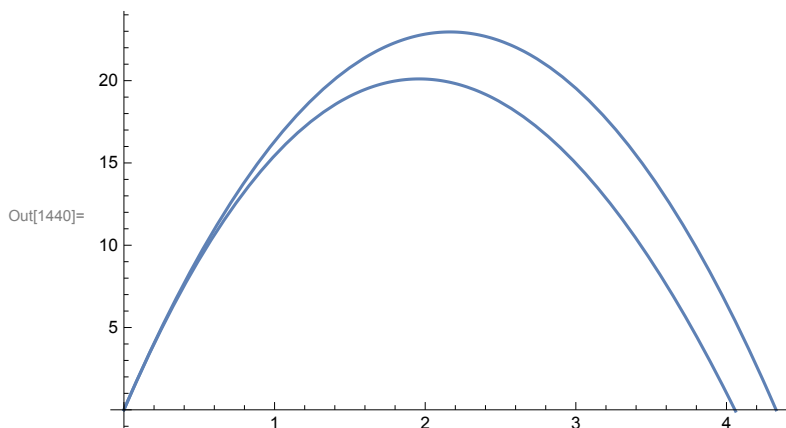
$$x(t) = \frac{v_0 \cos \theta}{k} (1 - e^{-kt/m})$$
$$y(t) = -\frac{mgt}{k} + \frac{k v_0 \sin \theta + g}{k^2} (1 - e^{-kt/m})$$

We can investigate the nature of these solutions by writing a short *Mathematica* code:

```
In[1429]:= Clear[v0, g, k, theta, x, y, xf, yf, m]
theta = pi/4; v0 = 30; k = 0.1; g = 9.8; m = 1;
(* x[t] and y[t] compute positions of the projectile with no friction *)
x[t_] := v0 Cos[theta] t
y[t_] := v0 Sin[theta] t - g t^2/2
(* xf[t] and yf[t] compute the position
of the projectile including frictional effects *)
xf[t_] := (v0 Cos[theta] / k) (1 - Exp[-k t / m])
yf[t_] := -m g t / k + ((k v0 Sin[theta] + g) / k^2) (1 - Exp[-k t / m])
(* the following is a slightly more advanced Mathematica code
designed to stop iterating once a specific condition is achieved,
here, we stop iterating once the projectile reaches the ground. The
variable "time" represents the value of t when y<0. *)
time = Catch[Do[If[yf[t] < 0, Throw[t]], {t, 0, 5, 0.01}]];
Print["Time of flight with friction = ", time, " secs"]
Print["Time of flight with no friction = ", 2 v0 Sin[theta] / g, " secs"]
g1 = Plot[y[t], {t, 0, 2 v0 Sin[theta] / g}];
g2 = Plot[yf[t], {t, 0, time}];

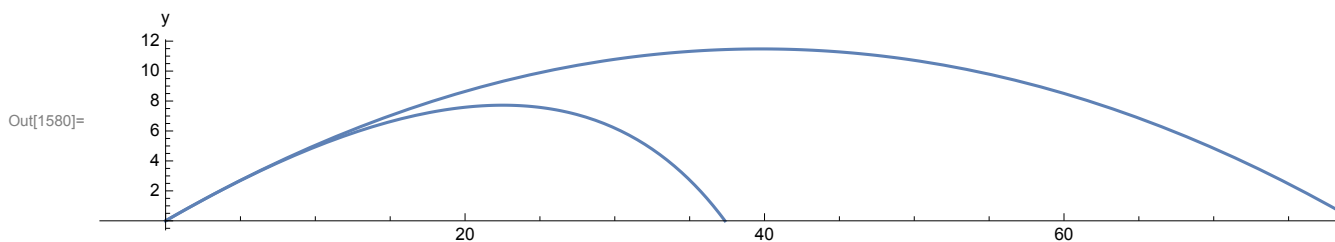
Show[g1, g2]
```

Time of flight with friction = 4.06 secs
 Time of flight with no friction = 4.32923 secs



Below is another version of the same program which shows another method for determining the time of flight within the program. In this version, we use the FindRoot function to solve the equation $yf[t]=0$ and determine the time of flight:

```
In[1570]:= Clear[m, g, k, v0,  $\theta$ , x, y, xf, yf, tfinal]
g = 9.8; v0 = 30; k = 0.5; m = 1;  $\theta$  = 30 Degree;
x[t_] := v0 Cos[ $\theta$ ] t
y[t_] := v0 Sin[ $\theta$ ] t - g t^2 / 2
xf[t_] := (m v0 Cos[ $\theta$ ] / k) (1 - Exp[-k t / m])
yf[t_] := - m g t / k + ((m k v0 Sin[ $\theta$ ] + m^2 g) / k^2) (1 - Exp[-k t / m])
tfinal = FindRoot[yf[t], {t, 2.5}];
tupper = t /. tfinal;
g1 = ParametricPlot[{x[t], y[t]}, {t, 0, 2 v0 Sin[ $\theta$ ] / g}];
g2 = ParametricPlot[{xf[t], yf[t]}, {t, 0, t /. tfinal}];
Show[g1, g2, AxesLabel -> {"x", "y"}, PlotRange -> All]
xfinal = xf[t] /. tfinal;
Print["Time of flight = ", tupper, "(s), Range = ", xfinal, "(m)"]
```



Time of flight = 2.53822(s), Range = 37.3561(m)

You can vary the initial angle and show that the maximum range occurs for an angle other than 45° . This version of the code introduces you to the “/.” command, known as the “slash dot” or “replace

all” command.