PHYS 314
HOMEWORK #2

Due : Feb. 1, 2017

1. We have been discussing the concept of invariance. In special relativity, the quantity that is invariant is :

$$(\Delta x)^2 - c^2\,(\Delta t)^2$$

where Δx is the spatial separation of two events and Δt is the temporal separation. If the same events are observed by observers in two different reference frames (call one the primed and the other the unprimed frame), invariance means :

$$(\Delta x)^2 - c^2\,(\Delta t)^2 = (\Delta x')^2 - c^2\,(\Delta t')^2$$

where the primes refer to the rocket frame and the laboratory is the unprimed frame.

a) You observe two events A and B which simultaneously in your lab frame at opposite ends of your lab bench which is 5 m long. Would an observer moving at 0.8 c with respect to your reference frame observe these events to be simultaneous? Use the equations to demonstrate why not? Determine the numerical value of the temporal separation observered in the rocket frame. (Remember to take length contraction into account; you may need to review your notes from modern for this problem).

2. a) Consider the two matrices :

$$A = \begin{pmatrix} 2 & 3 \\ -2 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 1 \\ -1 & 5 \end{pmatrix}$$

compute AB and BA. Do these by hand; you may use *Mathematica* to verify your results, but make sure you show your work.

b) Consider the matrices :

$$C = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ 3 & -1 & 2 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -2 \\ 4 & 1 & -3 \end{pmatrix}$$

Compute CD and DC by hand. You may use *Mathematica* to verify your results, but show your work in computing these products.

3. Find a non-trivial 2 x2 matrix R (i.e., not the identity matrix) such that $R^6 = I$. Show work and/or explain your reasoning.

4. Verify that the matrix :

$$\begin{pmatrix} \text{Cos}[\theta] & \text{Sin}[\theta] \\ -\text{Sin}[\theta] & \text{Cos}[\theta] \end{pmatrix}$$

is an orthogonal matrix (where the inverse = the transpose).

5. For this problem, refer to Fig. 1 - 4 (a) from the text. Consider the position vector **P** = {1, 2, 3}. Call α, β, γ, the angles between **P** and, respectively, the x, y and z axes. Compute the direction cosines (the cos of the angle between **P** and each of the coordinate axes) and then verify eq. (1.10) from the text.