PHYS 314
HOMEWORK #4

Due : 17 Feb. 2017

1. Using the text' s notation, start with equations 2.44 and 2.45 in the text and show all the intermediate steps to derive equations 2.49 and 2.50. (10)

2. Using the text' s notation, complete the intermediate steps to derive equation 2.55 in the text. (10)

3. A mass m moves in a circular orbit of radius r_o under the influence of a central force whose potential is -k m / r^n. The total effective potential of the mass is

$$U = \frac{-\,k\,m}{r^n} + \frac{L^2}{2\,m\,r^2}$$

where L is the constant angular momentum. For what values of n will the circular orbit be stable under small oscillations (that means the mass oscillates about the circular orbit).

The next two questions are variable mass problems, the first likely less complex than the second. Please work together on these (if you do, write the names of all group members on your homework; these are problems worth struggling with, so please don't try to find the answers on line or in texts).

4. A spaceship of surface area A and initial mass m_o and initial speed v_o moves through an interstellar dust cloud of density ρ. As the ship moves through the dust cloud, the dust sticks to the ship and adds to the ship's mass, but exerts no force on the ship.

a) Explain why the rate at which mass accumulates on the ship is (5)

$$\frac{dm}{dt} = \rho\,A\,v$$

b) Can you use the conservation of energy in this problem? Explain your answer. Is there any conservation law you can use in this problem?(5)

c) Find an expression for the velocity of the spaceship as a function of time. (15)

5. A raindrop falling through a cloud accretes matter as it falls. Assume the drop is always spherical and accretes matter according to :

$$\frac{dm}{dt} = 4\,\pi\,r^2\,k\,v$$

where r is the instantaneous radius of the drop, k is a constant, and v is the instantaneous speed of the drop. If the drop starts from rest when it is very small (say r approaching 0), show that the acceleration is constant and equal to g/7. (Note: dr/dt is the rate of increase of the radius of the drop; do not confuse this with the velocity of the drop. You should derive a differential equation which will require you to employ physical reasoning to solve.) (25)