

## Notes on the derivation in Sec. 2.2

For much of the rest of the semester, we will be concerned with counting the number of states accessible to a system. In section 2.1, we encountered the terms *microstate*, *macrostate* and *multiplicity*. It is important that you learn these terms well and thoroughly, as we will be using them and referencing them extensively.

In the first section, we learned how to determine the number of microstates and macrostates of simple systems. In section 2.2, we will extend these analyses to enable us to investigate interacting systems. The author does a good job of explaining (using the dot and line notation on the top of p. 55) how to calculate the number of microstates available to a system of  $N$  oscillators sharing a total of  $q$  units of energy.

In the last paragraph on p. 54, the author demonstrates how to determine the microstates available to a system of 3 oscillators, but what might not be sufficiently clear is that he is considering four separate cases: the case where 0 units of energy are distributed among the 3 oscillators, the case where 1 unit of energy is distributed across the oscillators, and the cases where two and then three units of energy are distributed among the three oscillators.

Thus, there is only 1 microstate in which the total energy is zero--namely the state in which all oscillators have zero energy.

There are 3 ways the oscillators can have a total of 1 unit of energy; there are 6 ways in which the oscillators can have a total of 2 units of energy, and there are 10 ways that the oscillators can have a total of 3 units of energy.

My concern is that the presentation of all these microstates in one table might lead to the confusion that the 20 microstates shown all belong to the same system.

Let's use these results to verify eq. 2.9 on p. 55:

$$\Omega(N, q) = \binom{q + N - 1}{q} = \frac{(q + N - 1)!}{q! (N - 1)!}$$

where  $N$  is the number of oscillators and  $q$  is the total energy of the system. Using the values from the table :

$$\Omega(3, 0) = \frac{(0 + 3 - 1)!}{0! \ 2!} = 1 \quad (\text{remember } 0! = 1)$$

$$\Omega(3, 1) = \frac{(1 + 3 - 1)!}{1! \ 2!} = 3$$

$$\Omega(3, 2) = \frac{(2 + 3 - 1)!}{2! \ 2!} = 6$$

$$\Omega(3, 3) = \frac{(3 + 3 - 1)!}{3! \ 2!} = 10$$

As you can imagine, the values of  $N$  and  $q$  become quite large (Avogadro's number large), so we will be computing factorials of very, very large numbers. Section 2.4 in the text shows us how to use Stirling's Approximation to calculate these factorials.